Statistical searches for low signal-to-noise helioseismic oscillations

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Outline

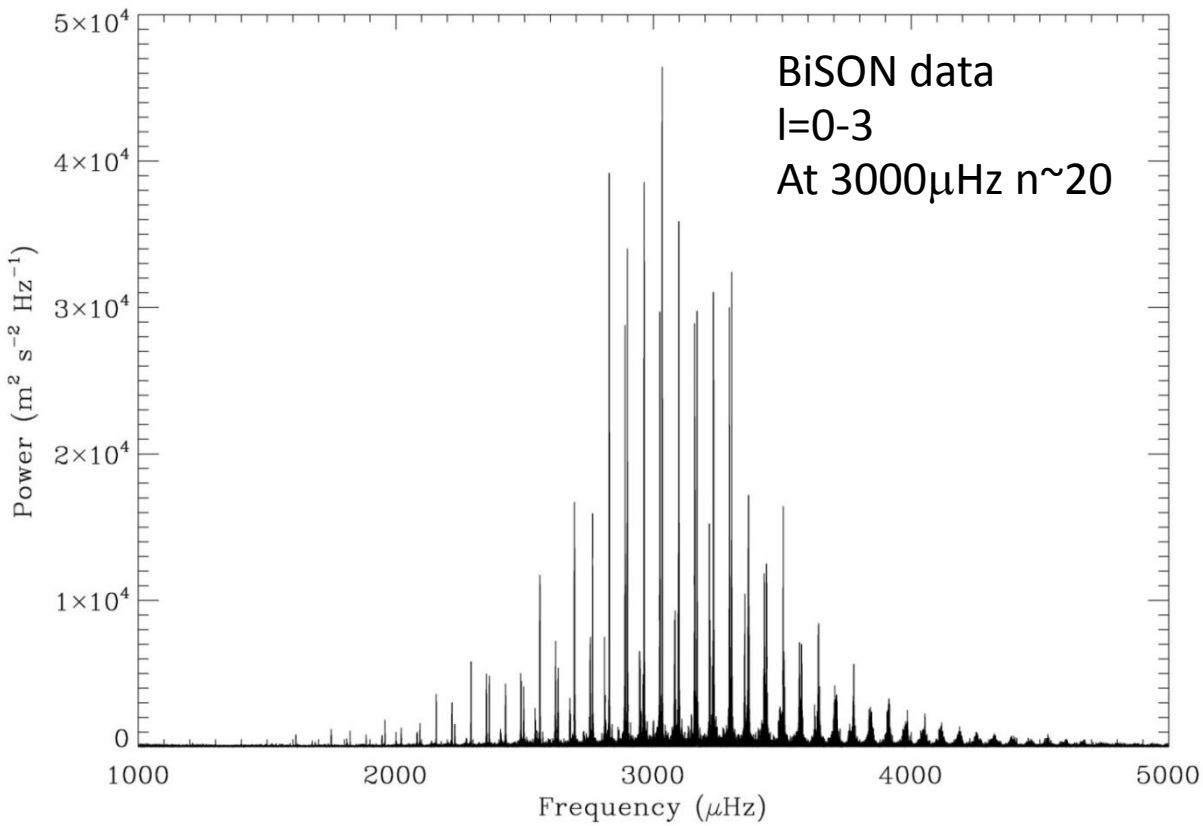
- Motivation – why bother looking
- Frequentist approach
- Bayesian approach
- Periodogram of a periodogram

Spaceinn – WP4 - Helioseismology

- ”Develop advanced statistical tools to extract the common information of contemporary datasets.”

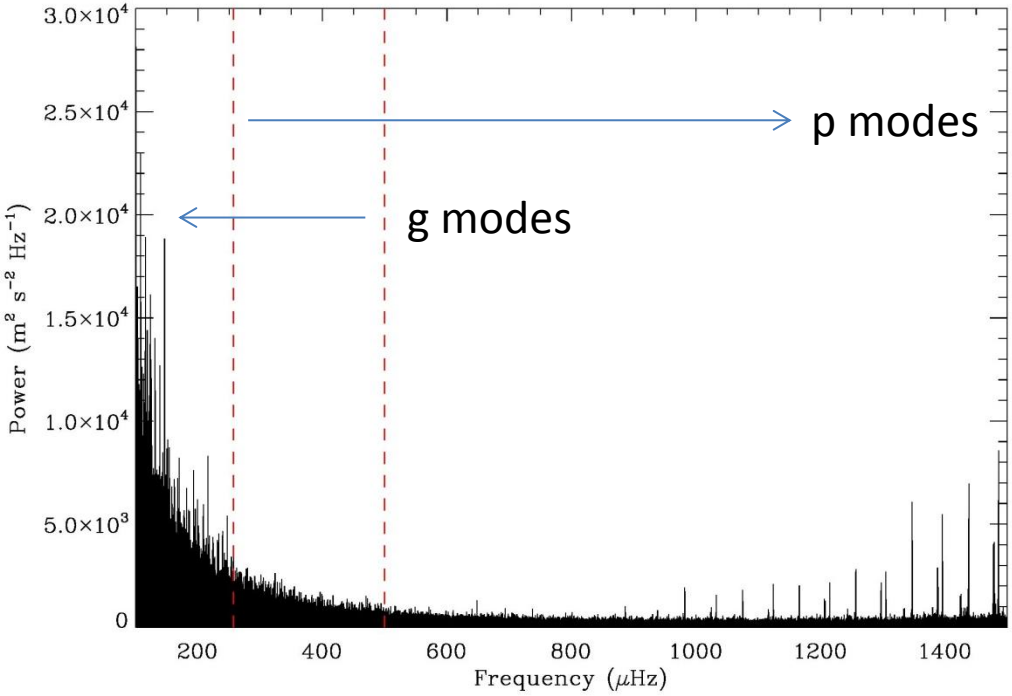
Motivation

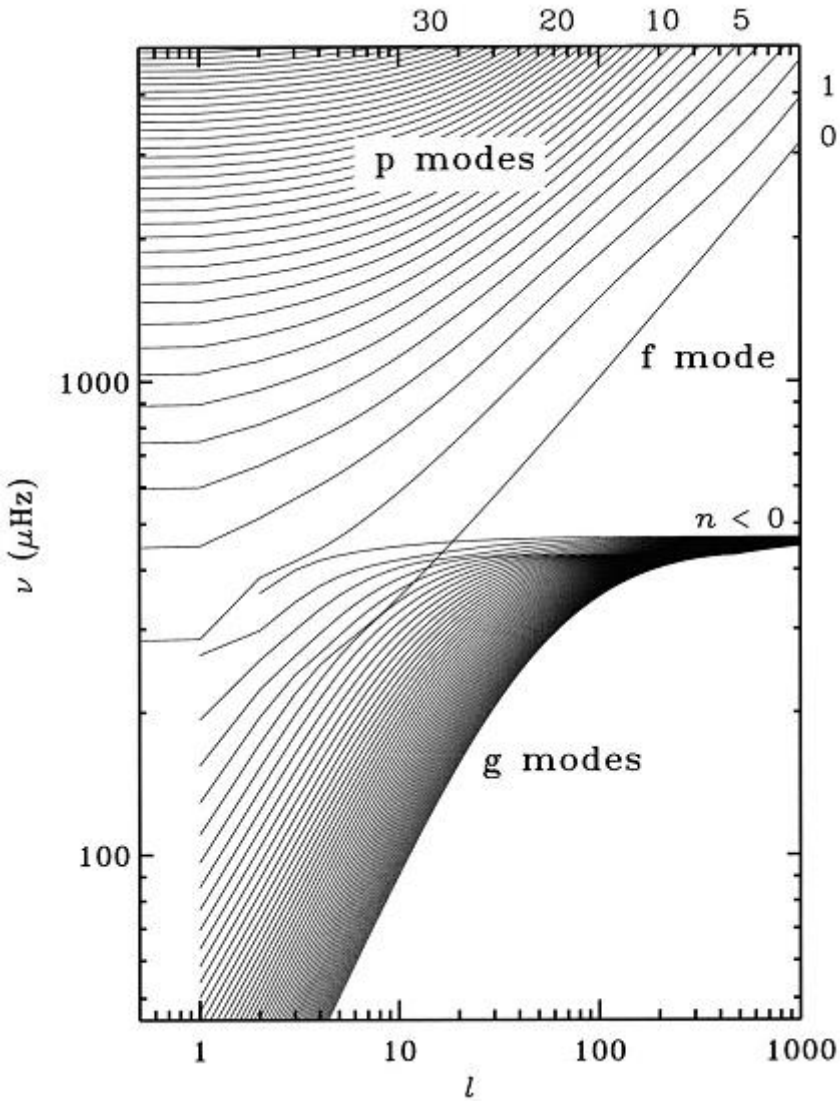
- Using Sun's natural internal oscillations to probe the solar interior



Motivation

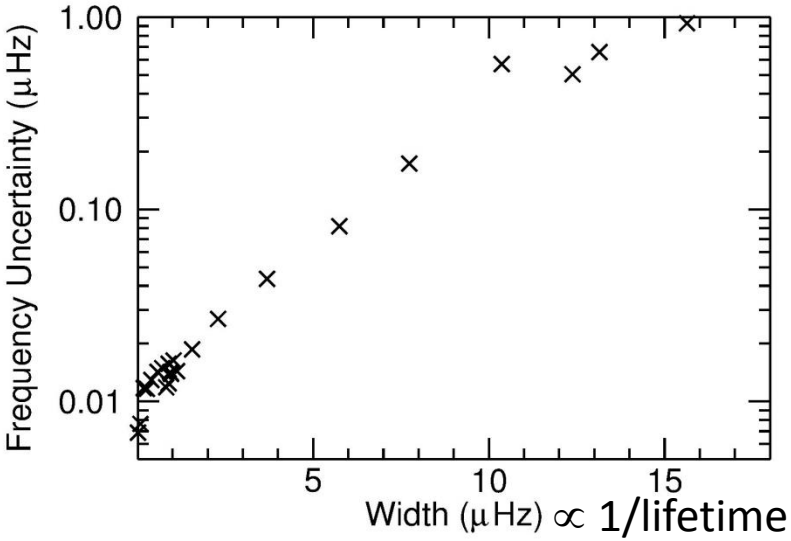
- At low frequencies we see:
  - Lower amplitude modes,
  - Higher noise.





From

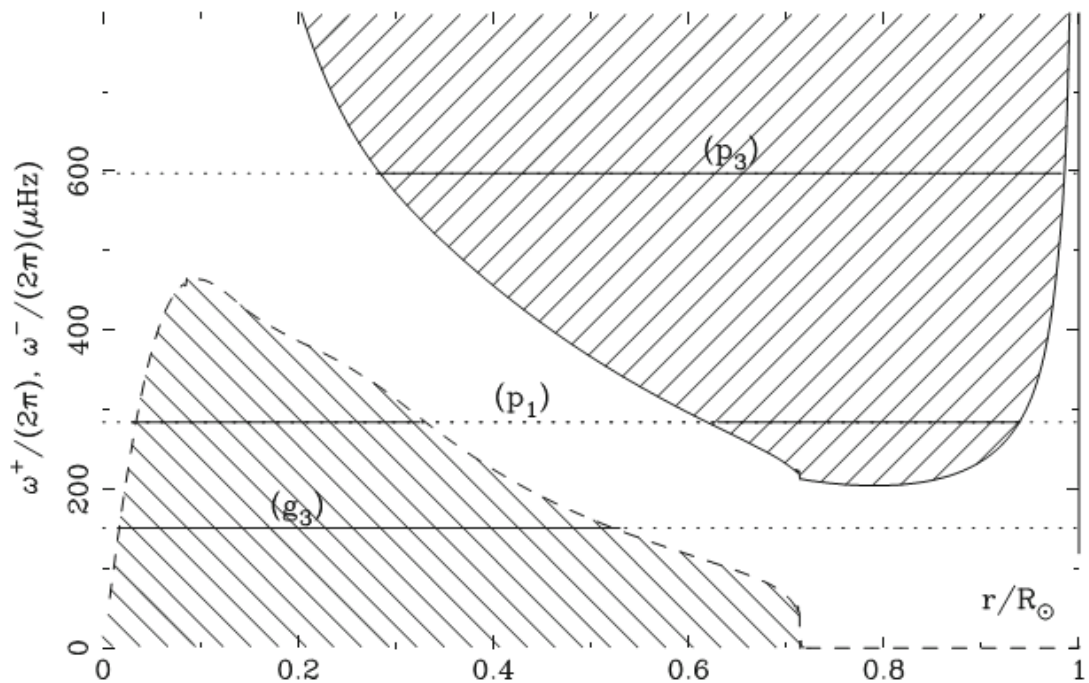
Motivation



Decreasing frequency

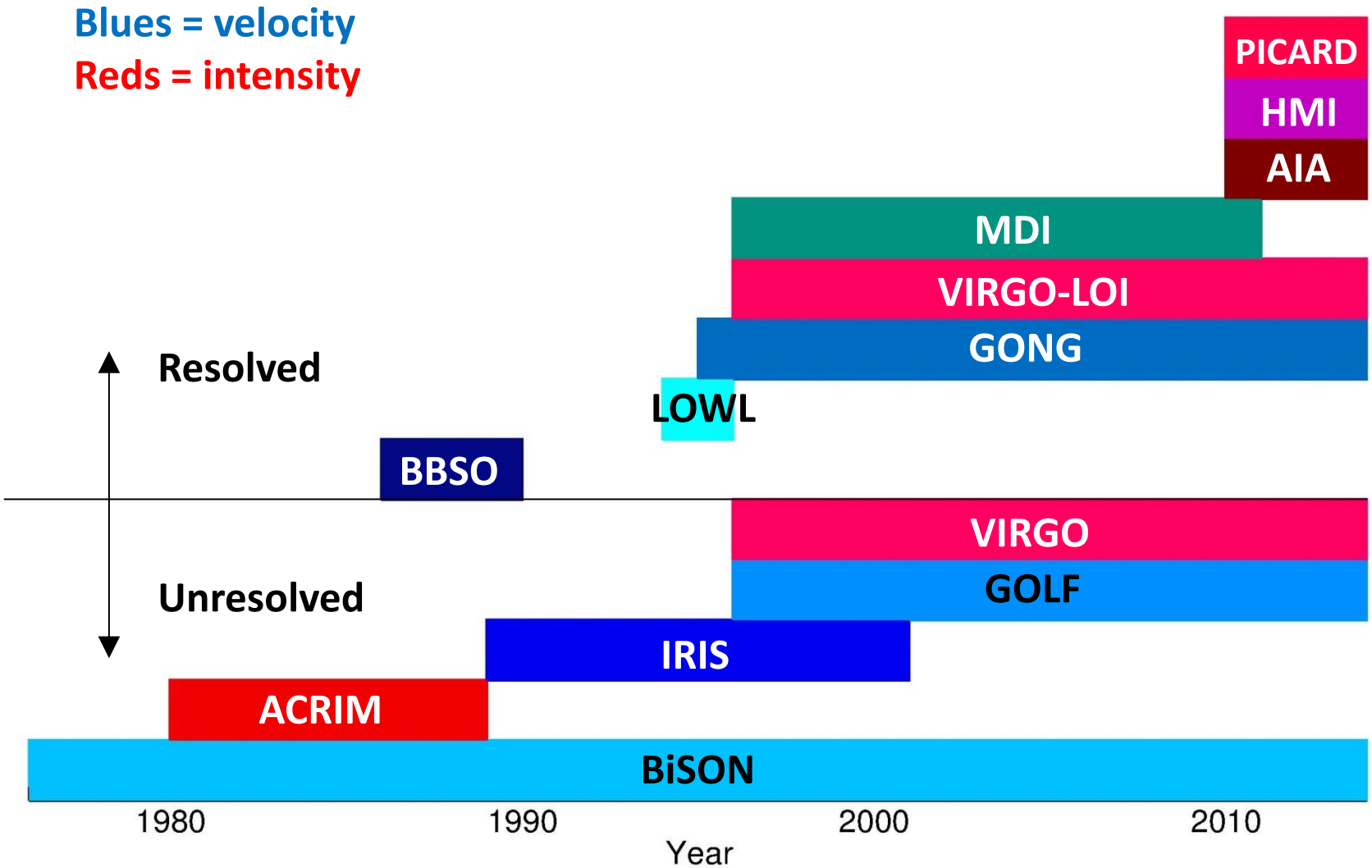
- Frequencies of low-frequency p modes can be obtained more accurately.

- Gravity modes far more sensitive to solar core.



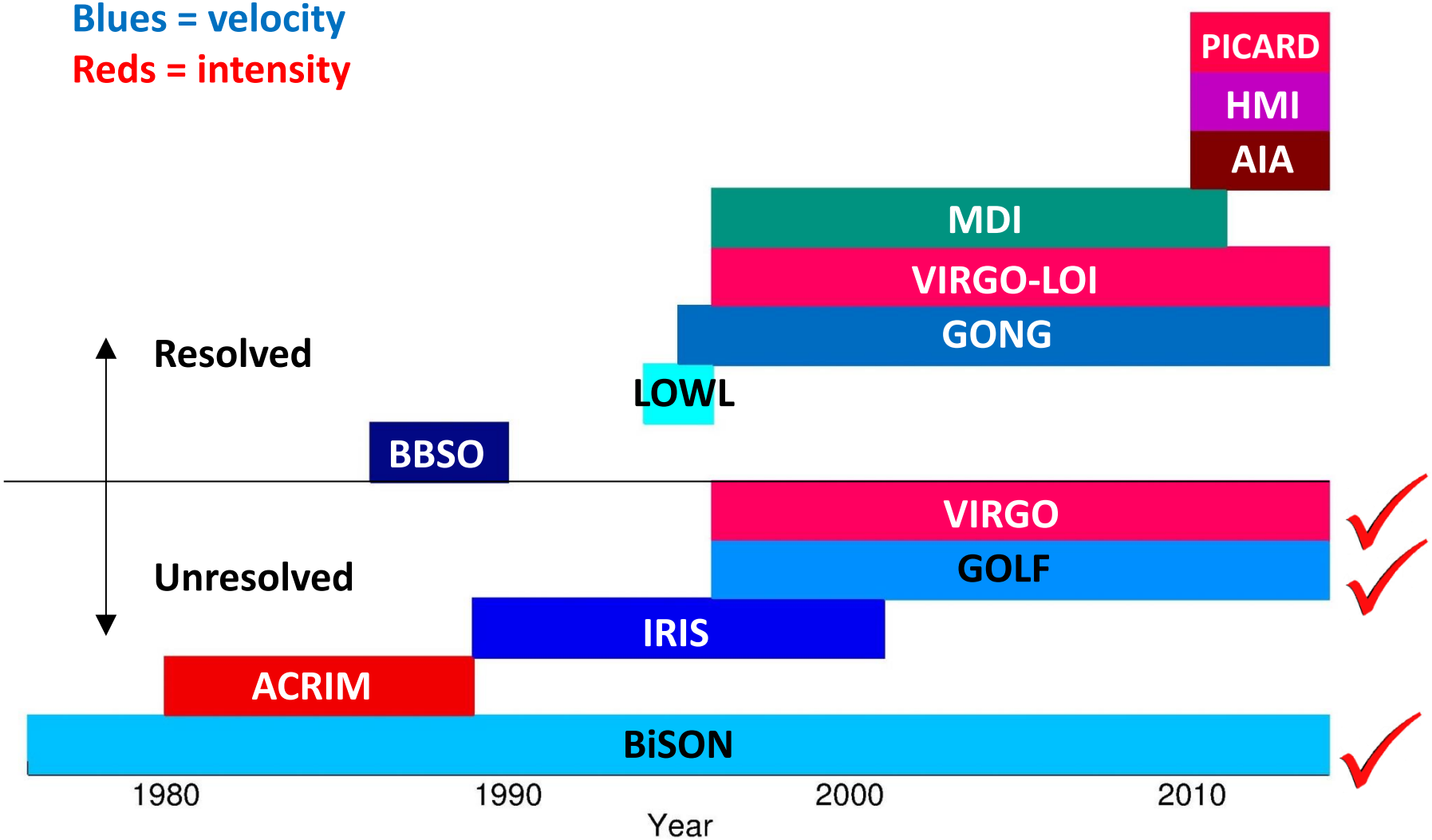
Appourchaux at al.

Data available



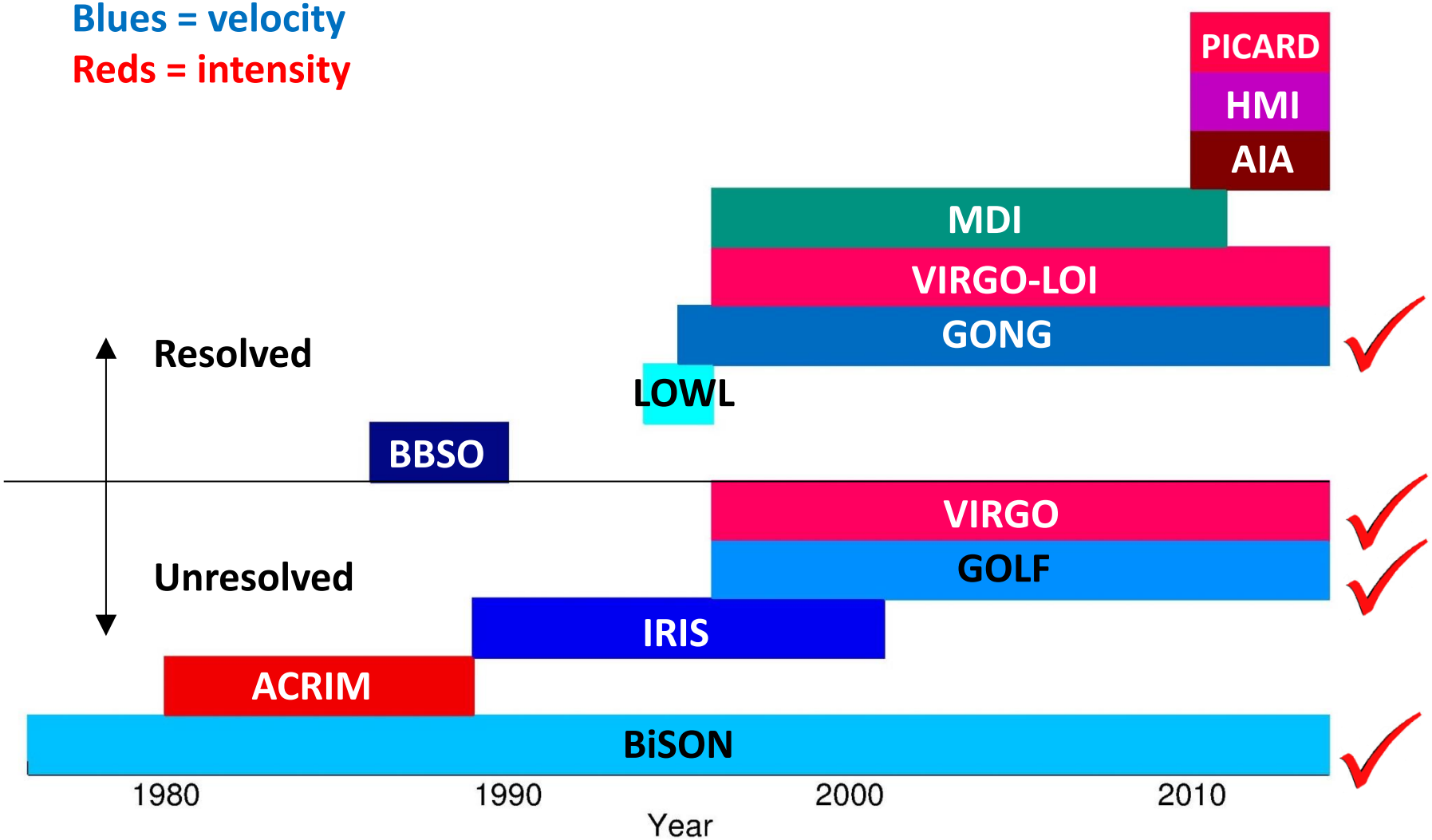
Data available

Blues = velocity
Reds = intensity



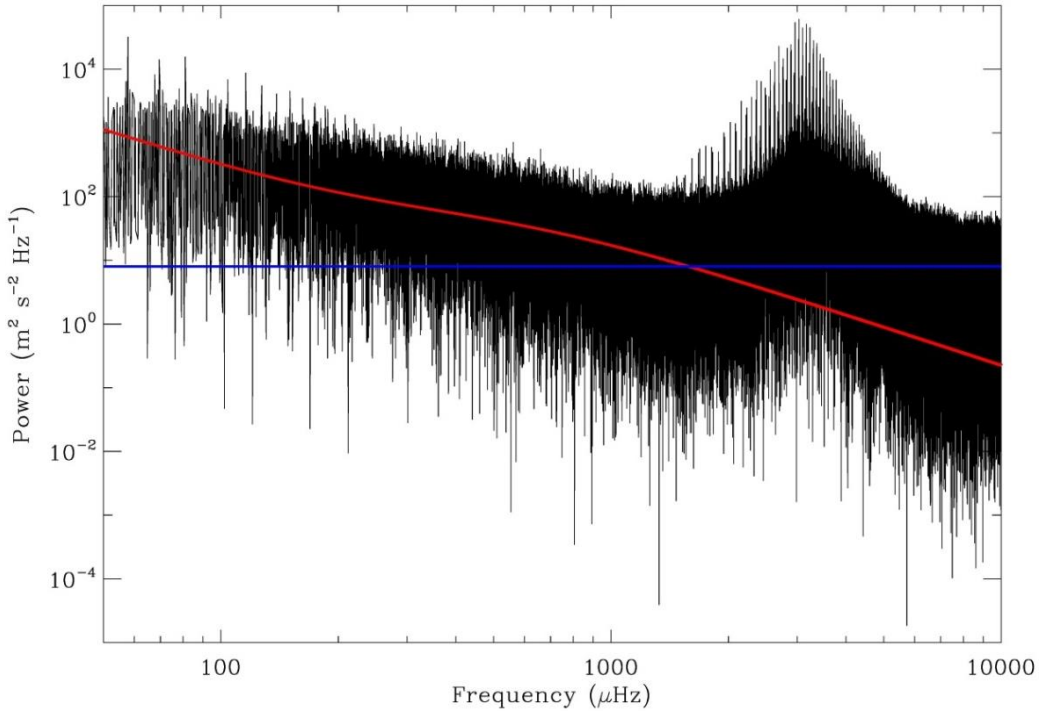
Data available

Blues = velocity
Reds = intensity



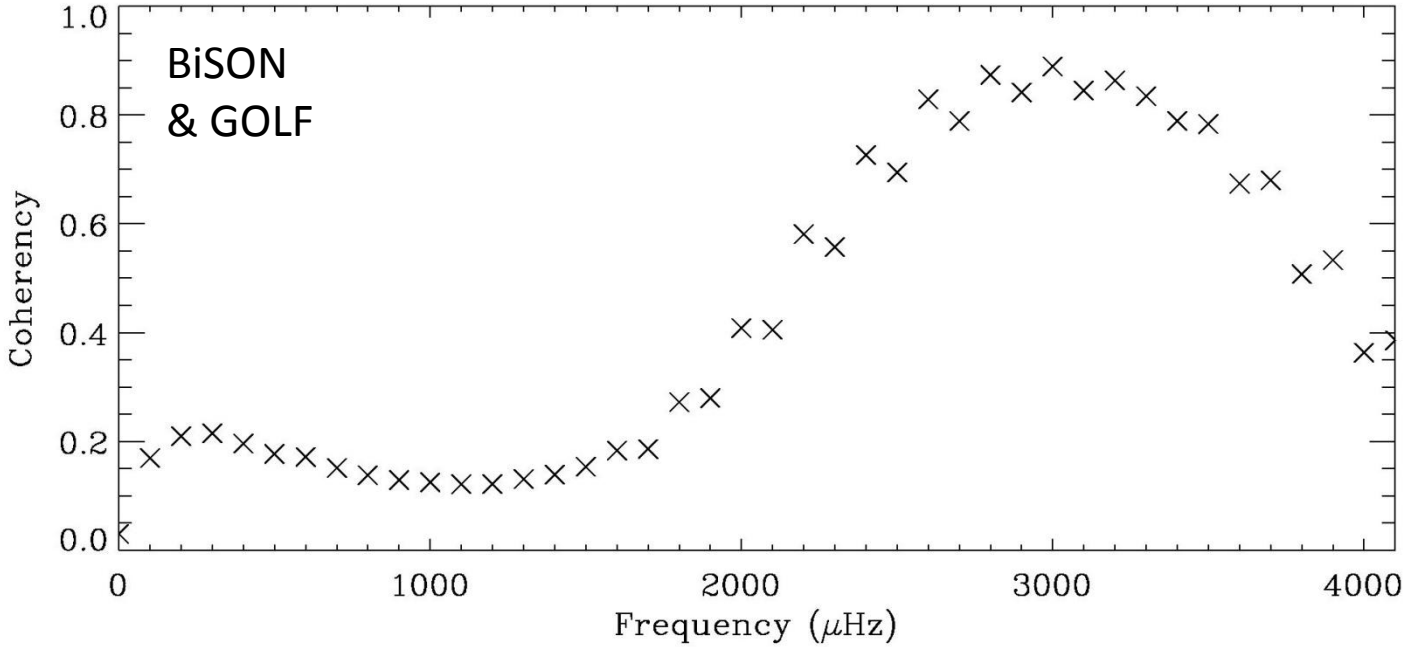
Contemporaneous data

- Main Aim: Emphasise coherent signal.
  - Coherent signal: oscillations.
  - Coherent noise: solar noise.
  - Incoherent noise: instrument, atmospheric.



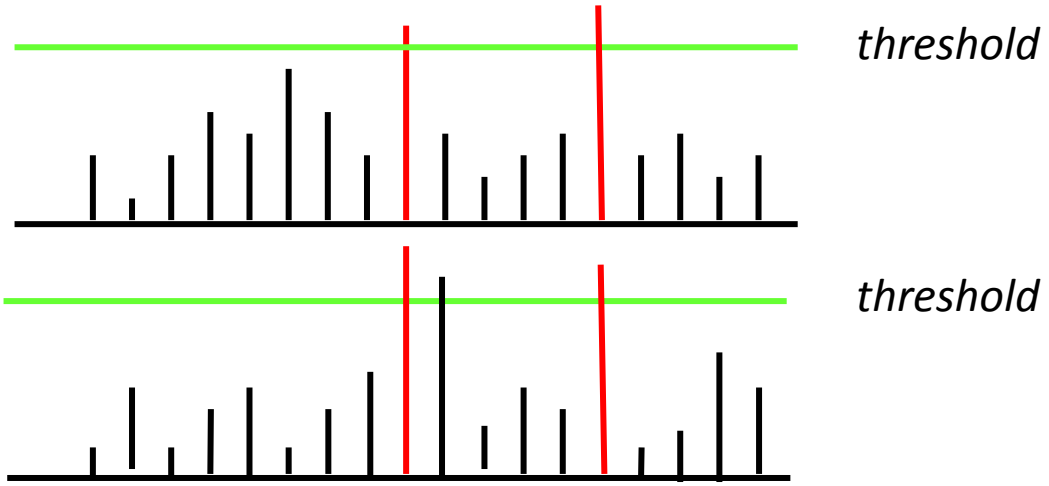
Common noise

- Solar noise will be common to data from two instruments.
- Proper allowance must be made for the level of noise common to the two sets of data.



Joint probability

- Allows searches for coincidences in contemporaneous data.
- Calculate probability of observing these coincidences in noise.
- Search for concentrations of power that lie significantly above the background noise level.



Frequentist approach

$$p = \sum_{n=0}^{\infty} \frac{1}{(n!)^2} \frac{\{\alpha[1+k(1-\alpha)+\alpha]\}^{2n}}{2^{2n}\sigma_a^{4n+4}[\alpha^2+\alpha(k-1)-k]^{4n+2}}$$
$$\times \frac{1}{4A_1A_2}\left[\sum_{m=0}^{n} \frac{n!r_1^{2(n-m)}}{(n-m)!A_1^m}\right]\left[\sum_{q=0}^{n} \frac{n!r_2^{2(n-q)}}{(n-q)!A_2^q}\right]$$
$$\times \exp\left\{-A_1r_1^2 - A_2r_2^2\right\},$$

where

$$A_1 = \frac{\alpha^2+[k(1-\alpha)+\alpha]^2}{2\sigma_a^2[\alpha^2+\alpha(k-1)-k]^2}$$

and

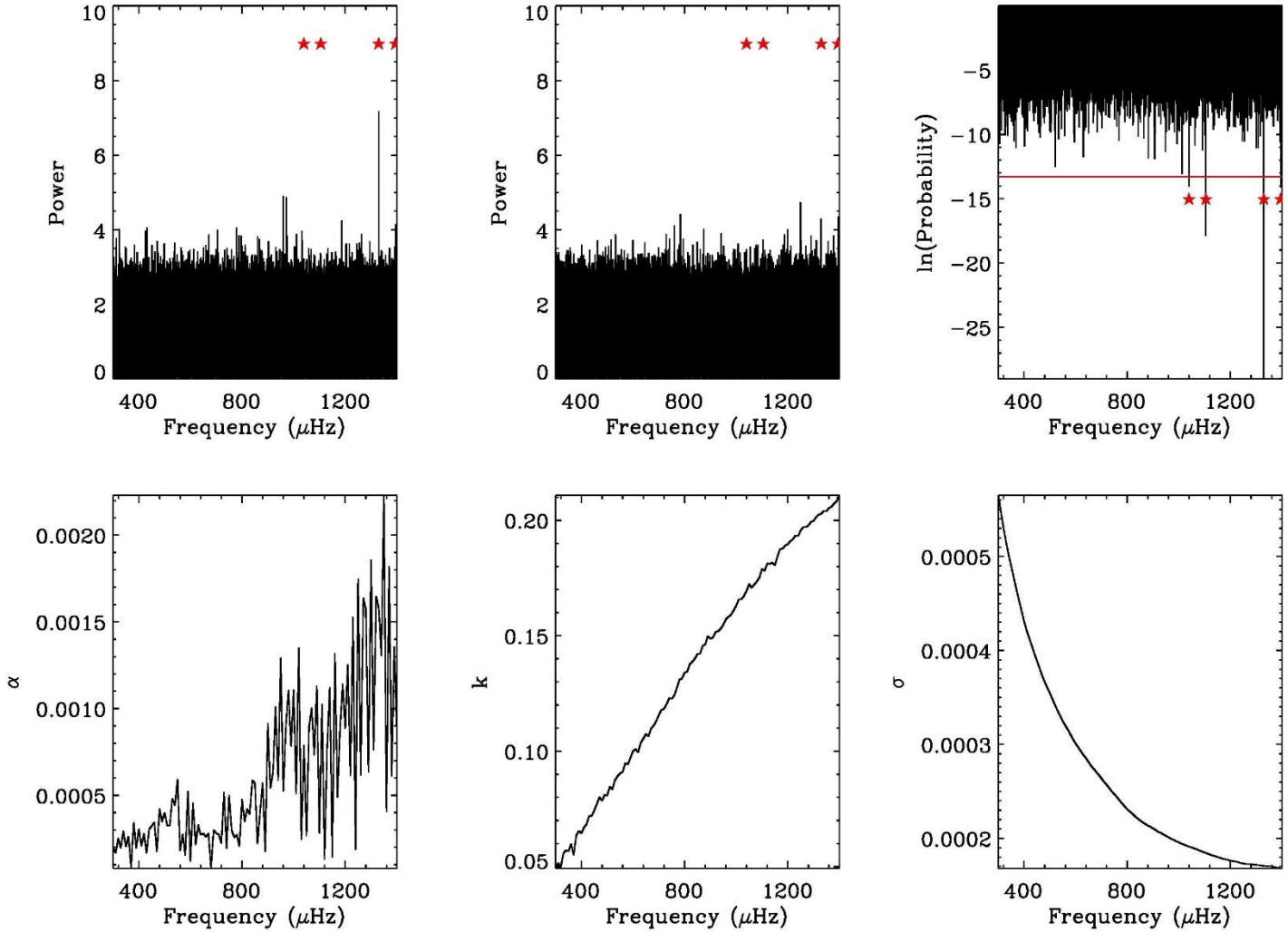
$$A_2 = \frac{(1+\alpha^2)}{2\sigma_a^2[\alpha^2+\alpha(k-1)-k]^2}.$$

p=joint probability of a prominent data point occurring at same frequency in 2 sets of not-independent data.

- r is AMPLITUDE of data point
- From data find
  - α: measure of proportion of common noise.
  - k and σ_a describe the variance of the data

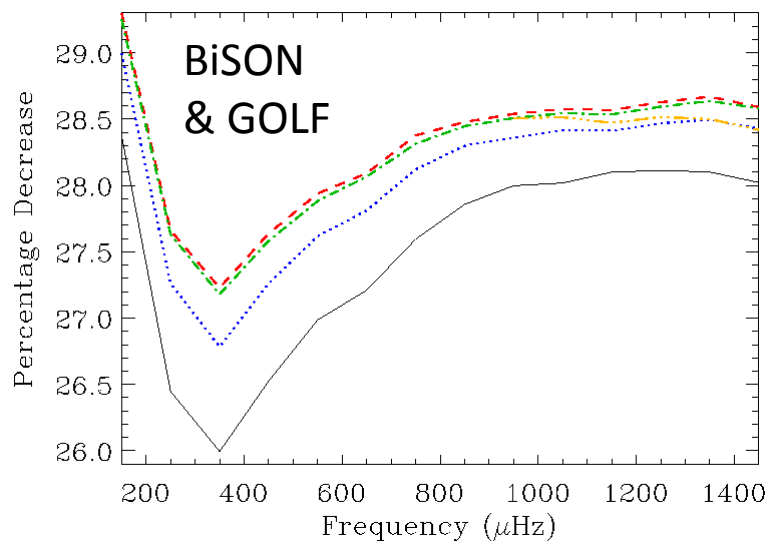
Examples from real data

BiSON GONG, l=2, m=2



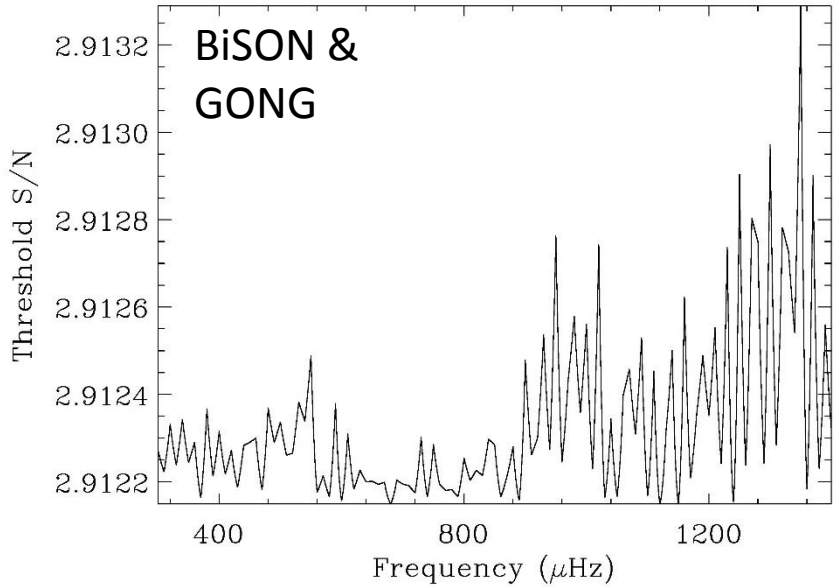
Reduction in amplitude thresholds

% decrease in amplitude detection threshold levels



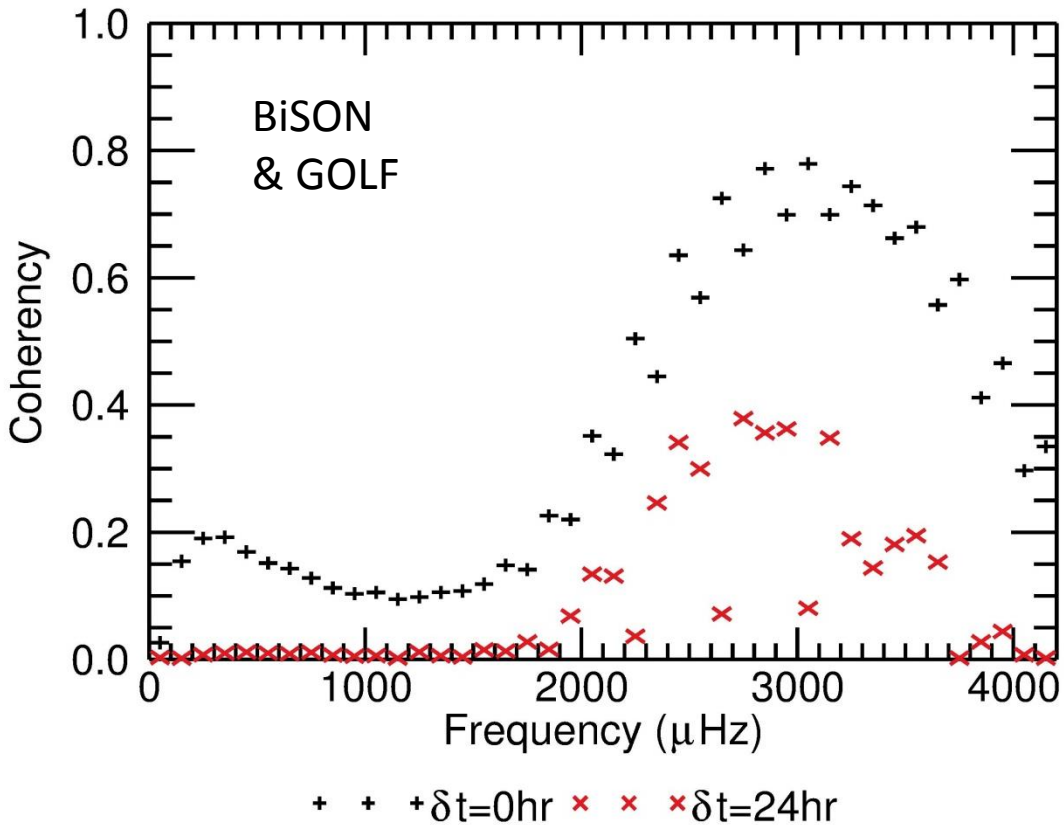
If coherency=0 reduced by √2

Single spike
2 consecutive spikes
2 or more spikes
2 rotationally split spikes
3 rotationally split spikes



NEAR-contemporaneous data

- Using data with start times that differ by 24hr can remove correlated noise.



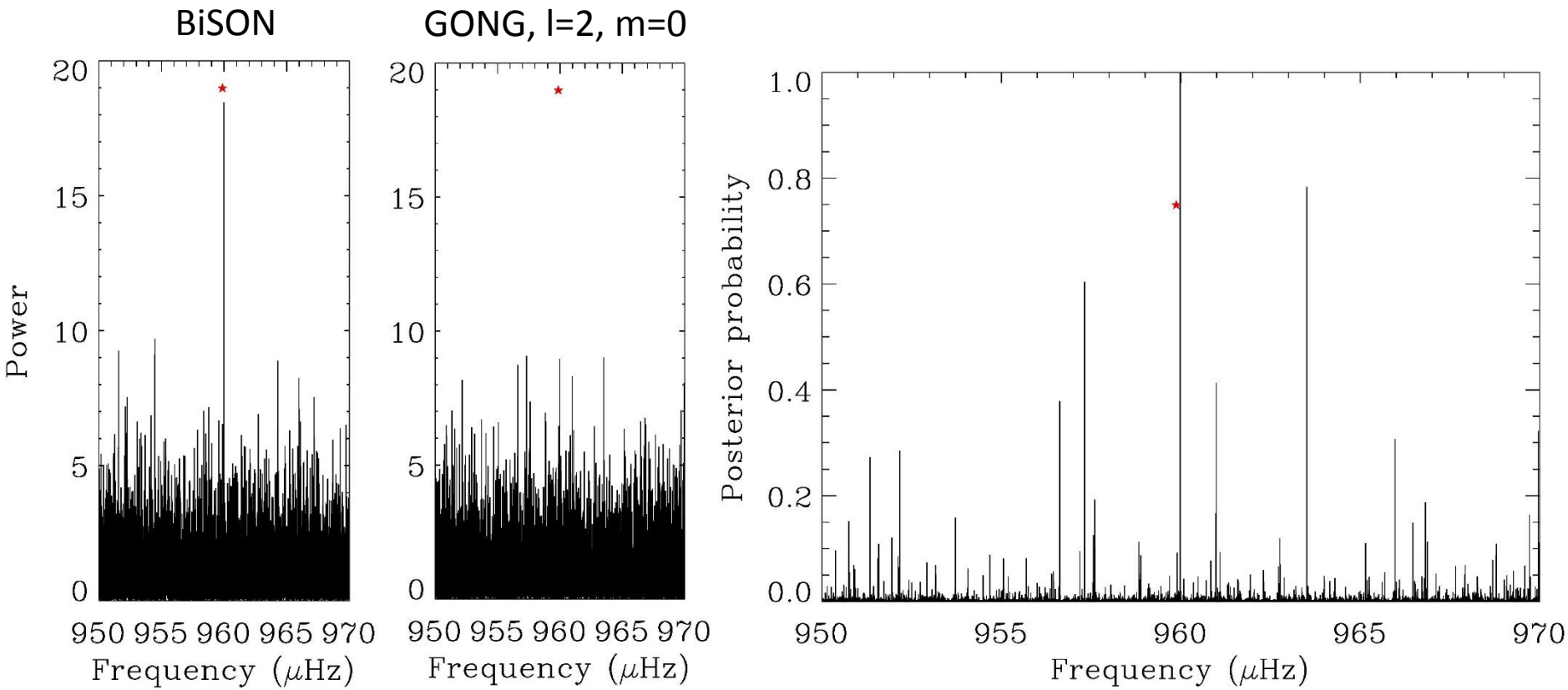
Bayesian statistics

- Posterior probability given by (e.g. Appourchaux et al., 2009):

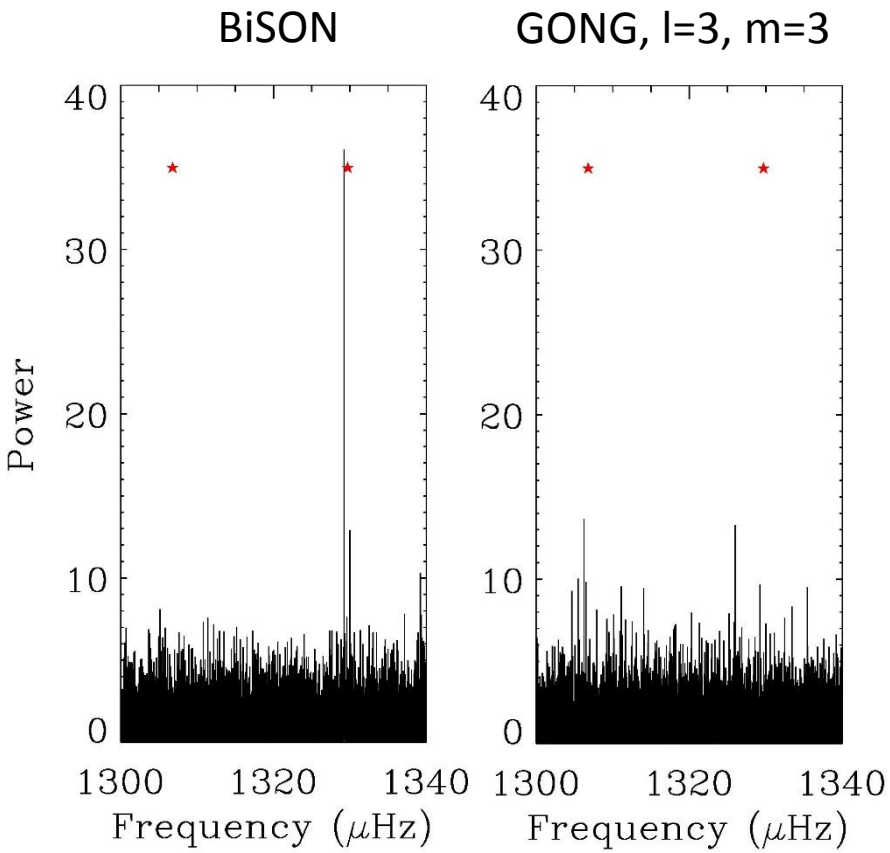
$$p(\mathrm{H0}|x) = \left[1 + \frac{(1-p_0)}{p_0}\frac{p(x|\mathrm{H1})}{p(x|\mathrm{H0})}\right]^{-1}$$

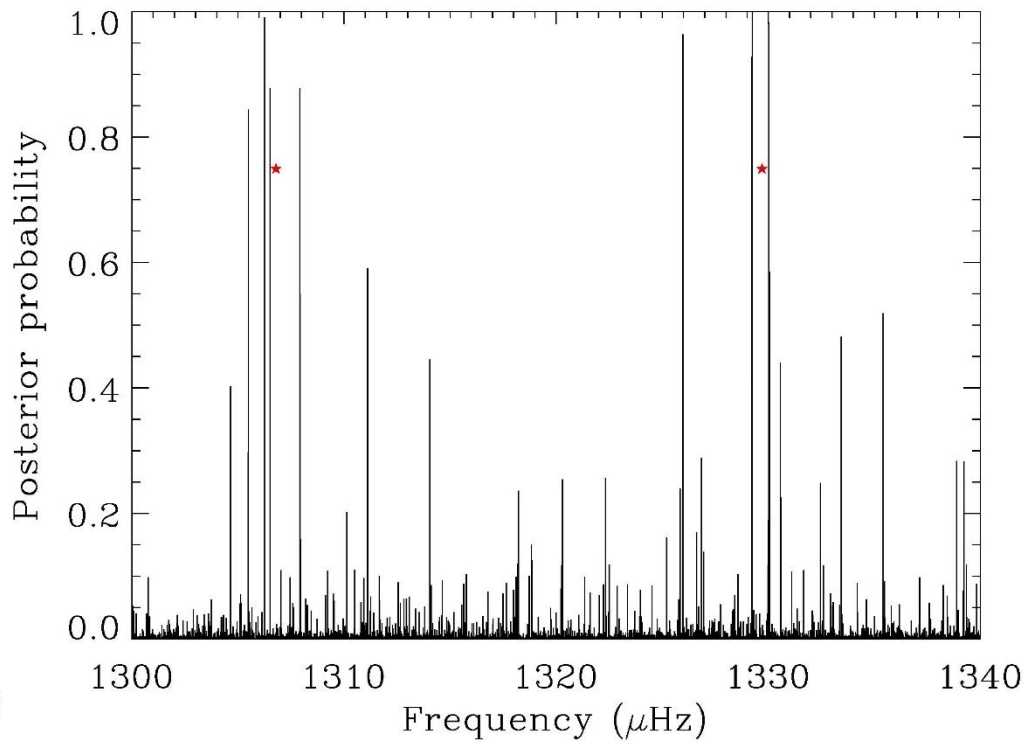
- p_0 = prior (simplest approach it $p_0 = 0.5$).
- $p(x|\mathrm{H0})$: probability that data point with height x observed in both spectra if H0 true.
- $p(x|\mathrm{H1})$: probability that data point with height x is observed in both spectra if H1 true.

Example result: l=2, n=5



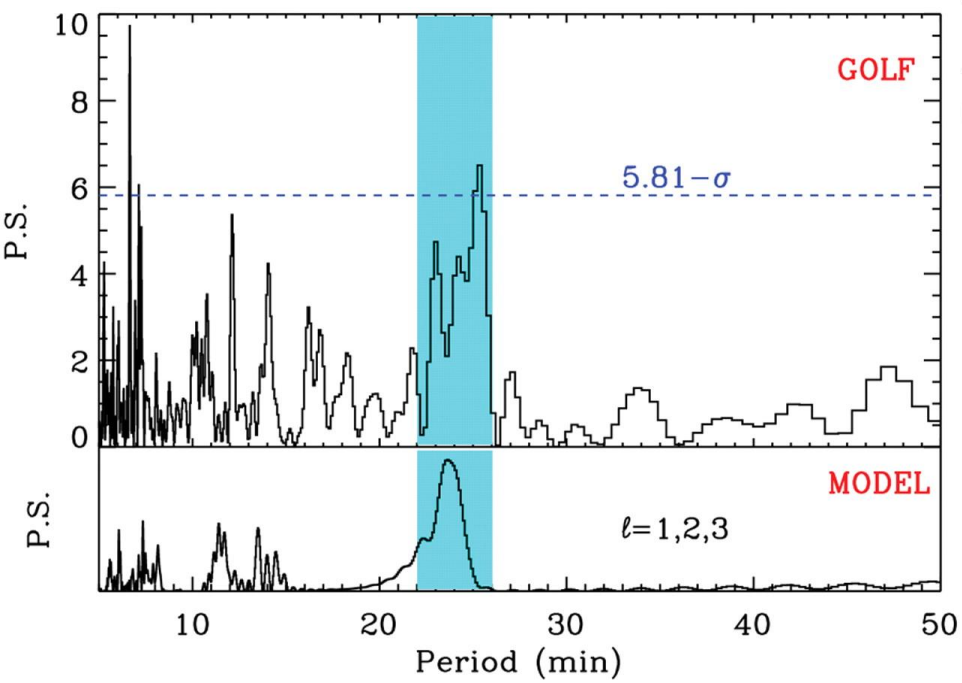
Example result: l=3, n=7

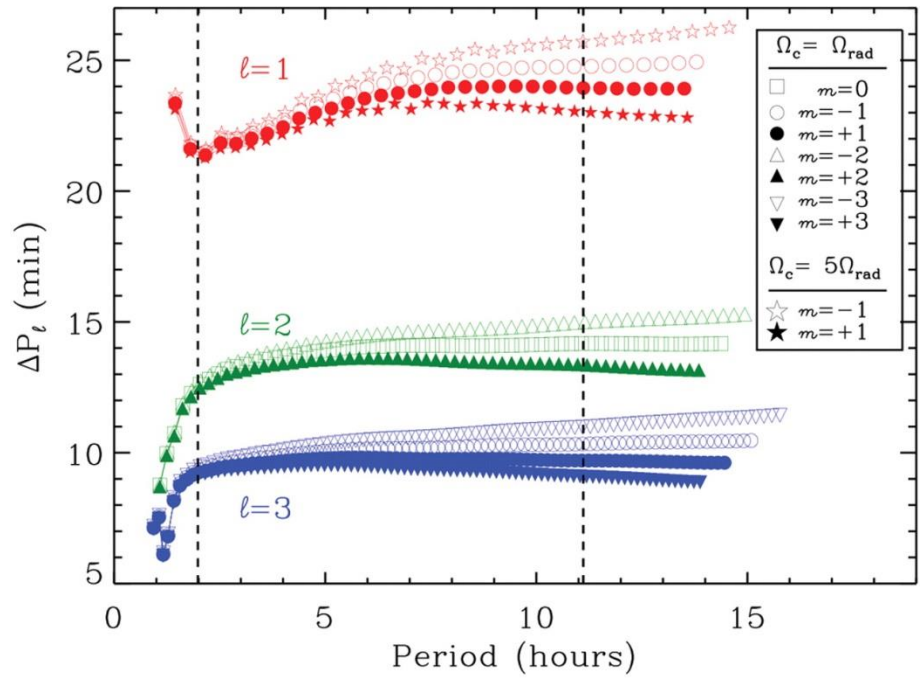




Periodogram of a periodogram

- Garcia et al., 2007, Science, 316, 1591



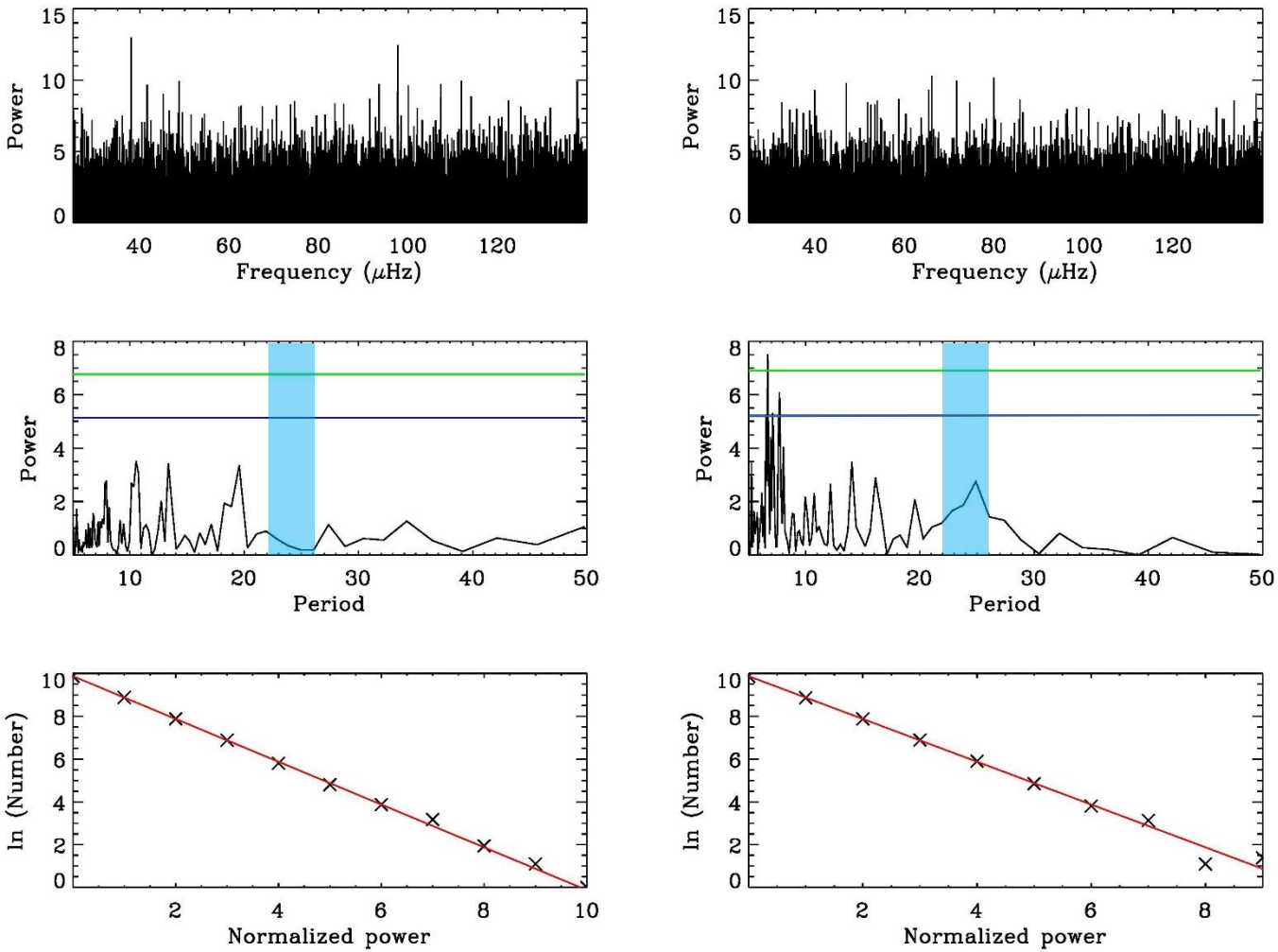


Joint periodogram

VIRGO GOLF

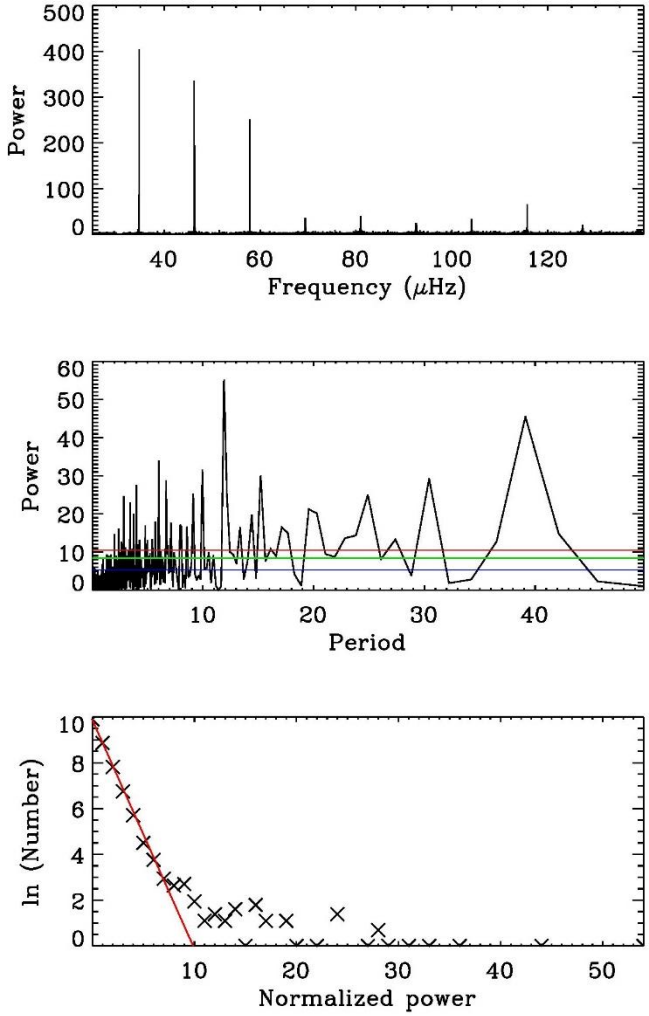
5.81σ

Combined 1 false

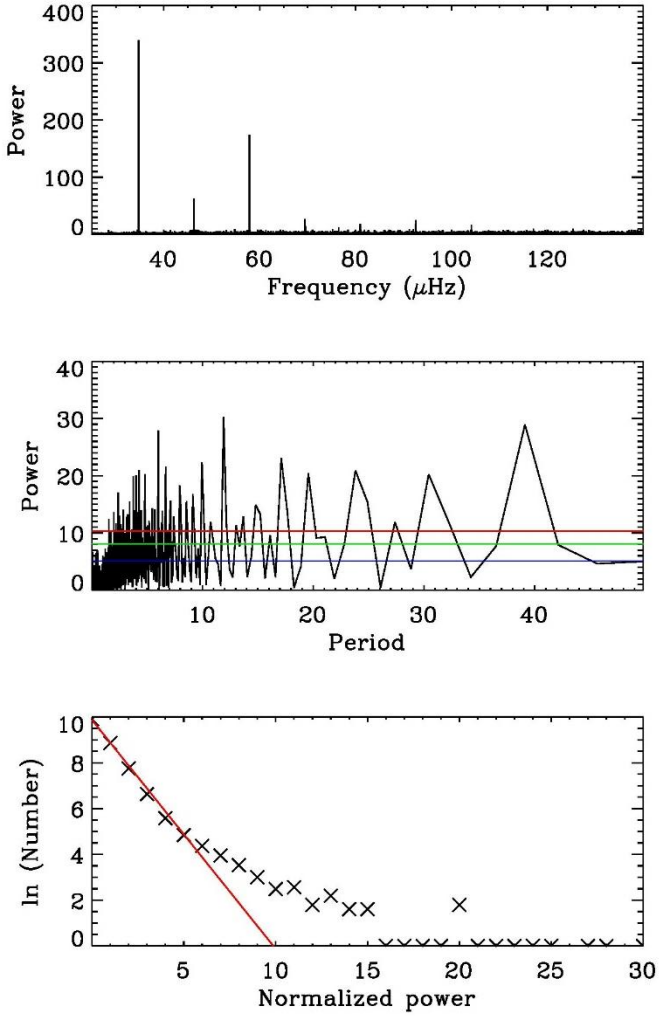


Joint periodogram – duty cycle

BiSON



GONG – l=2, m=0

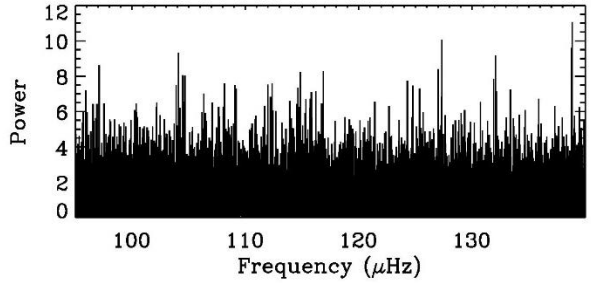


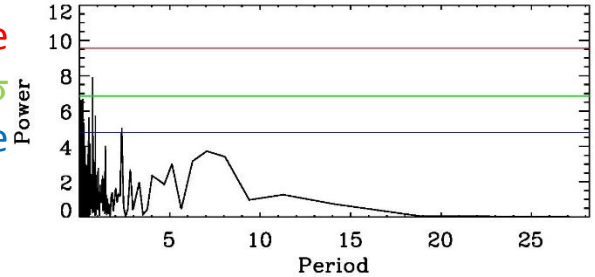
Joint periodogram - GONG

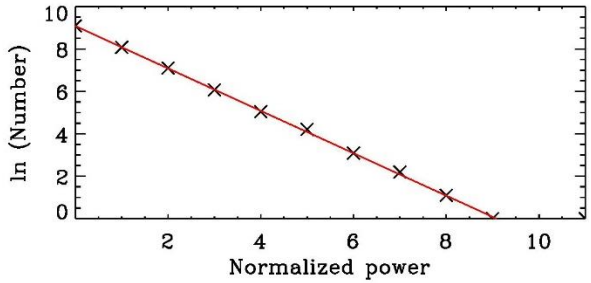
Start times separated by 24hr, Coherency=1%

Individual 1 false
5.81σ
Combined 1 false

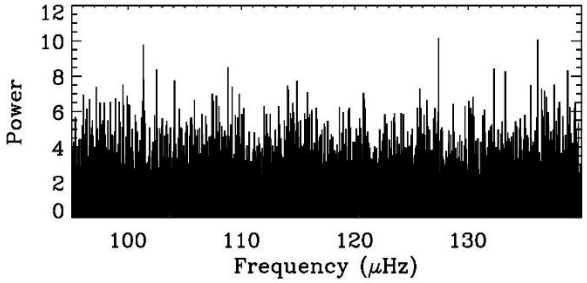
l=4, m=0

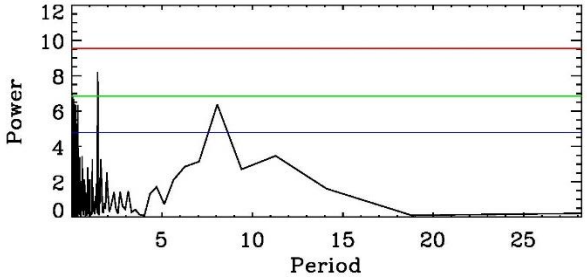


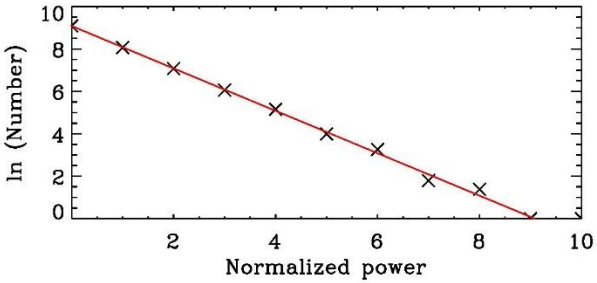


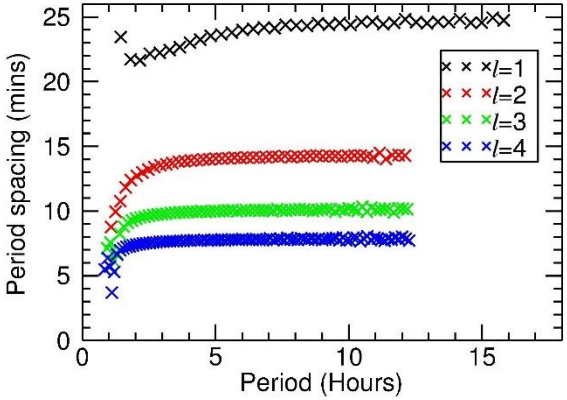


l=4, m=2









Summary

- Low frequency p modes and g modes are important for inversions of the solar interior.
  - BUT these modes have low-amplitudes.
  - Any detections need to be statistically robust.
  - Various tools have been developed.
- Some new low frequency p modes detected.
- Set thresholds on amplitudes of g modes.
- Future aims:
  - Incorporate SDO data.
  - Comparisons of 3 data sets.
  - Joint Bayesian analysis of periodogram of periodogram.