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ORIGINAL ARTICLE

Numerical analysis of flow over side weir

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Abstract In the present work the alternative concept of an elementary discharge coefficient along the longitudinal side weir was studied using an oblique weir. Five different angles (30°, 45°, 60°, 75°, and 90°) in the flow direction (inclined to the left) with respect to the branch channel wall were examined. A numerical solution of two ordinary differential equations was proposed for discharge and flow depth conducted with program using Euler's method and compared with experimental values. Results referred to the possibility of correlating water surface profile (W.S.P), water depth, and flow discharge numerically as well as the possibility of increasing flow discharge passing through the branch channel by an oblique side weir installation. Thus flow discharge can be increased with side weir angle. Maximum discharge occurred at an angle of 30° where the percent increase compared with that at 90° was 70%.

1. Introduction

Weirs are the hydraulic structures that have been used for flow measurement, diversion, regulation, energy dissipation and many other means. A side weir, installed in the channel to divert the flow, is used as a key structure in many hydraulic projects. (Borghei et al., 1999). Due to the use of various geometric and hydraulic shapes of the side weir, many research works have been conducted for different types of side weir.

Previous studies with many experimental investigations into rectangular channels such as Subramanya and Awasthy, 1972; Ramamurthy et al., 1980; Swamee et al., 1994; Bruce, 1995; Swamee et al., 1995; Singh et al., 1994; Jalili and Borghi, 1996; Huagao et al., 2007 and Mwafaq and Ahmed, 2011 deal with the discharge coefficient in subcritical flow and developed an equation for the discharge coefficient.

The hydraulic behavior of different types of weirs has been studied by Ramamurthy et al., 1978; Knight, 1989; Uyumaz, 1992; Nihat et al., 2011 and Vatankhah, 2012; they also investigated the discharge coefficient for sharp crested side weir and developed an equation for discharge coefficient.

Many theoretical studies deal with discharge coefficient (C_d) and water surface profile (W.S.P) for flow over the side weir. Smith (1973) used computer programing for flow over the side weir; Ramamurthy et al., 1986 used polynomial equation to examine the side weir discharge coefficient; Swamee et al., 1994; Ojha and Subbaiah, 1997; Honar, 2004 and Ahmed, 2011 studied the discharge coefficient in subcritical flow using the Runge Kutta method and compared the results with experimental finding. Yilmaz (2011, 2002), Yilmaz et al. (2004) investigated the lateral weir flow numerically and using curve fitting analysis as well as transition effects respectively

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developed an equation to calculate the side weir discharge coefficient. Ahmed et al. (2013) used Simulink programing to simulate flow over the side weir for water surface profile and discharge coefficient. Mahmodiniaa et al., 2012 investigated water surface profile and discharge coefficient using numerically simulated FLUENT program.

Flow over the side weir in supercritical flow was investigated theoretically and experimentally by Rao and Pillai, 2008 using chi-squared test and developed the effect of supercritical flow discharge coefficient and calculated the water surface profile. The flow over an oblique weir and side weir to calculate weir discharge coefficient and develop an equation for these calculations was studied by Samani, 2010; Borghi et al., 2003; Nalder, 2004; Swamee et al., 2011 and Azza, 2012.

The present work deals with studying energy principle and energy loss concept and is an alternative of elementary discharge coefficient along the side weir with different angles of side weir with respect to side channel wall in the flow direction (inclined to the left). A methodology based on the numerical solution of two ordinary differential equations using Euler's method was proposed for discharge and flow depth.

2. Dimensional analysis

Discharge coefficient (C_d) for an oblique side weir is influenced by the approaching velocity of flow v_1; the upstream depth of flow y_1; the height of side weir crest p; the length of side weir L; the channel width b; acceleration due to gravity g and the angle of side weir with respect to the branch channel θ, where;

$$C_d = f(v_1, y_1, p, L, b, g, \theta) \tag{1}$$

Dimensional analysis for the above parameters gives:

$$C_d = f_1\left(F_1, \frac{p}{y_1}, \frac{L}{b}, \theta\right) \tag{2}$$

where $F_1 = \frac{v_1}{\sqrt{gy_1}}$ is the approaching Froude number.

By applying the experimental data for the above parameters, using the Statistical Package for the Social Sciences (SPSS, version 17), the following equation was obtained:

$$C_d = 1.275 - 0.612F_1 - 0.522\frac{p}{y_1} + 0.028\frac{L}{b} - 0.132\theta \tag{3}$$

$R^2 = 0.959$ and θ in radian.

3. Theory

The equation of spatially varied flow with decreasing discharge can be applied for flow over a side weir using the energy principle which assumed that the longitudinal component of velocity vector of spill flow at any section is equal to the average velocity of flow in the channel. Therefore the total energy per unit mass of water remaining in the channel is unaffected by the spill flow occurring, and apart from frictional losses the total energy of the flow in the main channel remains constant Rao and Pillai, 2008.

According to the energy principle, the slope of the water profile can be expressed as,

$$\frac{dy}{dx} = \frac{s_o - s_f - \frac{\alpha Q}{gA^2}\frac{dQ}{dx}}{1 - \frac{\alpha Q^2 T}{gA^3}} \tag{4}$$

in which T: top width of the water surface; s_f: average frictional slope; A: area of flow in the main channel; α: energy correction factor; dQ/dx: discharge per unit length over side weir; dy/dx: slope of the water surface profile over side weir; s_o: slope of the channel; Q: discharge in the main channel; g: acceleration due to gravity.

Considering the parameter dQ/dx discharge dQ through an elementary strip of length dx along the side weir, the following equation can be written as:

$$-\frac{dQ}{dx} = \frac{2}{3}C_d\sqrt{2g}(y - p)^{3/2} \tag{5}$$

For subcritical flow and rectangular section; the energy correction factor α can be written as 1, then using Manning's equation for rectangular channel of bed width b and combining Eqs. (4 and 5) yield the following equation:

$$\frac{dy}{dx} = \frac{s_o - \frac{Q^2 n^2}{b^2 y^{10/3}}\left(1 + \frac{2y}{b}\right)^{4/3} + \frac{2\sqrt{2}}{3}cd(y - p)^{3/2}\frac{Q}{b^2 y^2 \sqrt{g}}}{1 - \frac{Q^2}{gb^2 y^3}} \tag{6}$$

where n: Manning's roughness coefficient.

4. Experiments

When Froude number was less than 1, subcritical flow, type (b) conditions are satisfied in this case. Depth of flow at the upstream flow direction was less than that at the downstream section along the longitudinal side weir, Chow, 1959 Fig. 1. According to Azza, 2012, experiments were carried out in the hydraulic laboratory of the Water Resources Department, University of Mosul, Iraq. The rectangular flume was 10 m long, 30 cm wide, and 45 cm deep. The side channel was 15 cm wide, 30 cm deep, and 2 m long. The dimensions and positions are shown in Fig. 2.

The actual discharge (Q_{act}) of the main channel was measured using a standard rectangular sharp crested weir, installed

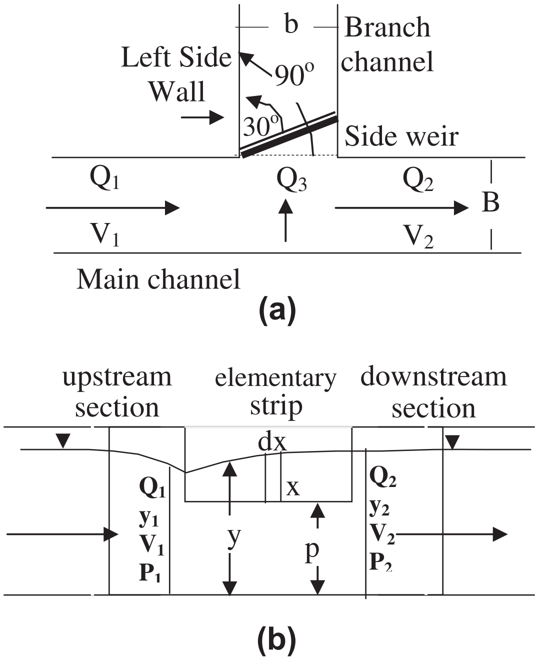

Figure 1 Definition sketch; (a) plan; and (b) elevation.

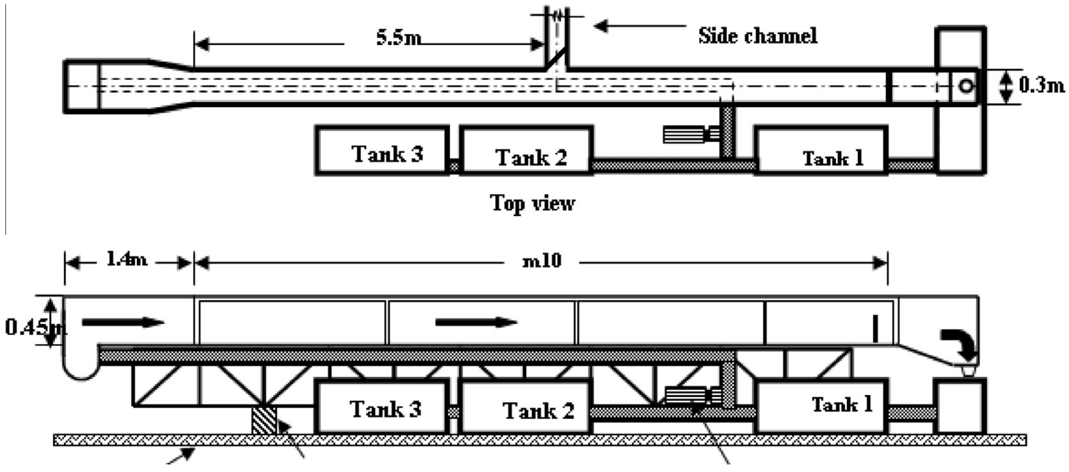

Figure 2 Laboratory flume.

Table 1 Range of parameters measured.

Angles with wall (θ)^o	Crest height (P) cm	$F1$	Length of side weir (L) cm	$Q1$ (L/s)	$Q2$ (L/s)	$Q3$ (L/s)
90	12	0.110	15	7.278	6.878	0.401
90	12	0.130	15	9.612	8.954	0.658
90	12	0.151	15	11.676	10.398	1.277
90	12	0.172	15	13.869	12.393	1.476
90	12	0.183	15	15.660	13.869	1.791
75	12	0.116	15.5	7.686	7.077	0.609
75	12	0.137	15.5	10.285	9.171	1.113
75	12	0.156	15.5	12.152	10.742	1.410
75	12	0.178	15.5	14.373	12.635	1.738
75	12	0.184	15.5	15.922	13.995	1.927
60	12	0.124	17.32	8.312	7.481	0.831
60	12	0.146	17.32	10.973	9.612	1.361
60	12	0.160	17.32	12.635	10.973	1.662
60	12	0.183	17.32	15.141	13.124	2.017
60	12	0.186	17.32	16.185	14.045	2.140
45	12	0.129	21.2	8.738	7.686	1.052
45	12	0.150	21.2	11.440	9.834	1.605
45	12	0.165	21.2	13.124	11.205	1.919
45	12	0.185	21.2	15.400	13.247	2.153
45	12	0.187	21.2	16.449	14.120	2.329
30	12	0.134	30	9.171	7.893	1.279
30	12	0.155	30	11.913	10.059	1.854
30	12	0.169	30	13.619	11.440	2.179
30	12	0.192	30	16.053	13.619	2.434
30	12	0.185	30	16.449	13.869	2.580

35 cm from the end of the main channel, with dimensions $15 \times 30 \times 1$ cm. The side weirs had dimensions of 12×(15, 15.53, 17.32, 21.2, 30)×1 cm which were installed at the entrance of the side channel with angles (30^o, 45^o, 60^o, 75^o, and 90^o) with respect to the side channel wall in the flow direction (inclined to the left).

Five different discharges ranged from 7.3 to 16.5 l/s, 25 runs were made. The range of various parameters is given in Table 1.

The main channel discharge was found using the following equation using volumetric calculations for standard rectangular sharp crested weir calibration to find discharge using the stage discharge relationship as follows:

$$Q_{\text{act}} = 0.58^{*}H^{1.5} \tag{7}$$

where Q: discharge (l/s), H: head over standard weir (cm).

The discharges calculated in this way represent the actual discharge (Q_{act}); by closing the side channel and measuring the head over the standard weir, upstream discharge (discharge before side weir, Q_1) can be calculated but when opening the side channel and measuring the head over standard weir, downstream discharge (discharge after side weir, Q_2) can be

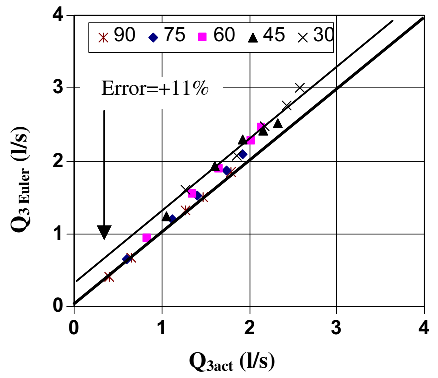

Figure 3 Comparison between actual discharge computed from Azza (2012) and those theoretical values computed in this work using Euler's method.

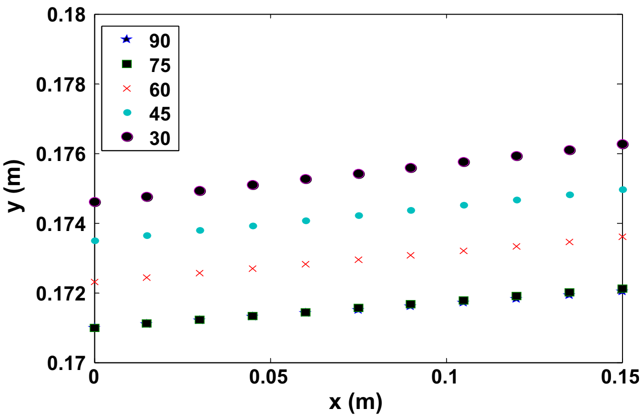

Figure 4 Water surface profile along the longitudinal side weir for data computed numerically using Euler's method using Matlab language.

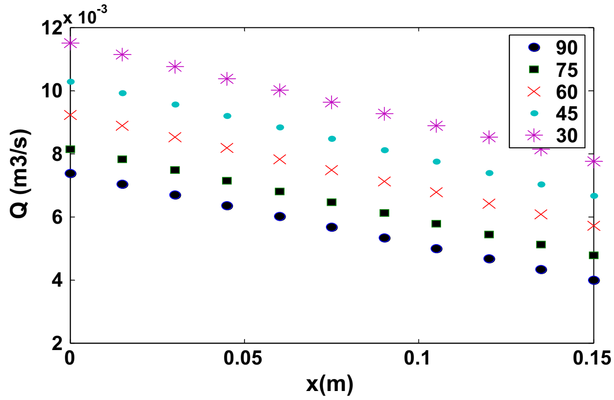

Figure 5 Flow discharge along the longitudinal direction of the side weir for data computed numerically using Euler's method using Matlab language.

calculated. Thus the actual discharge through the side channel can be calculated from the equation:

$$Q_3 = Q_1 - Q_2 \tag{8}$$

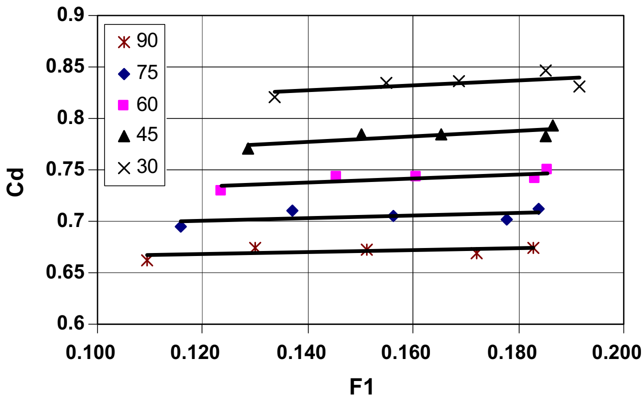

Figure 6 Correlation of discharge coefficient and approaching Froude number.

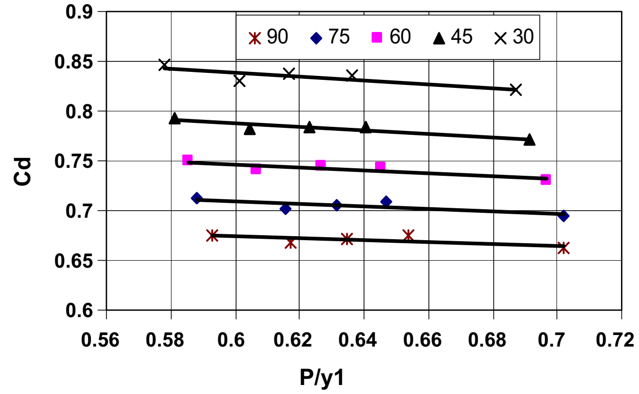

Figure 7 Correlation of discharge coefficient and relation of weir height and approaching water depth.

Variation of the water surface profile (W.S.P.) and flow discharge was measured and calculated along the longitudinal side weir.

5. Results

From experimental data of each run, Eqs. (5 and 6) can be solved by Euler's method from the following equation:

$$y_{(i+1)} = y_i + f(x_i, y_i)w \tag{9}$$

where

$y_{(i+1)}$: Downstream water depth slide
$y_{(i)}$: Upstream water depth slide
$w = x_{(i+1)} - x_i$

Eqs. (5 and 6) were solved using Matlab language V.R2010a for initial condition at $x = 0$; $y = y_1$; $Q = Q_1$ and the discharge coefficient value was calculated from Eq. (3). Values resulting from numerically solving Eqs. (5 and 6) represent the flow discharge and water surface profile (W.S.P) along the longitudinal x direction for the side weir crest length.

Fig. 3 shows a comparison of the observed discharge which was measured by Azza, 2012 and computed in the present work using Euler's method. It can be seen that the majority of the data points which fall in the error do not exceed 11%.

Experimental data collected were used in computer programing using Euler's method to find the water surface profile

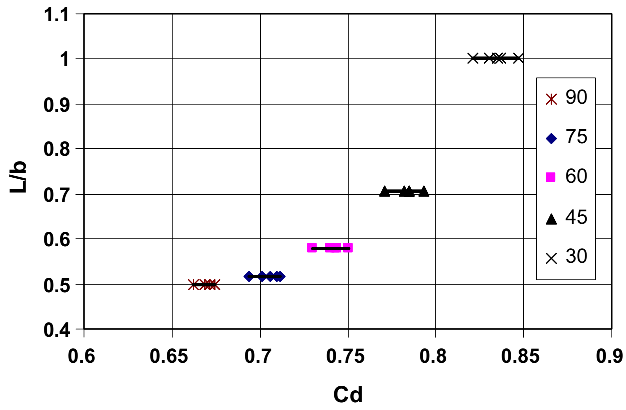

Figure 8 Correlation of discharge coefficient and crest length to channel width.

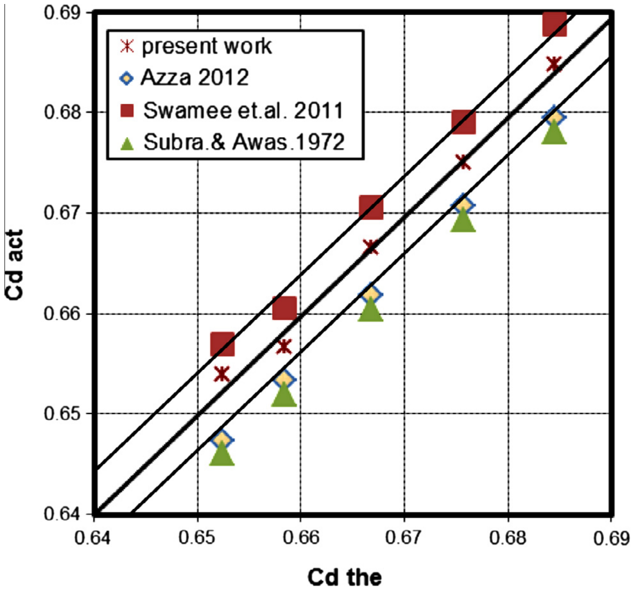

Figure 9 Correlation of discharge coefficient in the present work with other previous works at 90°.

(W.S.P) and compared with those observed experimentally. A longitudinal W.S.P plotted along the crest length of the side weir for different angles is shown in Fig. 4.

At all side weir angles the water surface profile increases along the longitudinal x direction from the upstream to the downstream side weir location, the case being identical Chow, 1959, type (b) conditions are satisfied when the depth of flow is greater than a critical depth at the entrance with subcritical flow in the weir section. Water depth increased when the side weir angle increased and the maximum water depth occurred at an angle of 30°.

Fig. 5 shows the computed discharge measured by Euler's method along the longitudinal direction of the side weir. It is shown that discharge decreased from upstream to downstream weir, at the same time flow discharge increased when the side weir angle increased and the maximum discharge occurred at a side weir angle of 30°, with an average percentage increase of 70% when compared to that at 90° weir angle.

This means the possibility of increasing the discharge by changing the side weir angle when other hydraulic parameters of side channel remain constant.

The coefficient of discharge computed from eq.3 is shown in Figs. 6–8. It is shown in Fig. 6 that C_d increased when approaching the Froude number and the side weir angle increased and maximum values occurred when the side weir angle was 30° with a 30% increase than that at 90°. In Fig. 7, C_d decreased when the relation of the weir height and the approaching water depth increased while the coefficient increased when the angle increased and maximum values occurred when the side weir angle was 30°. As shown in Fig. 8 C_d remains constant at all values of crest length to channel width, and that coefficient values increased when the side weir angle was 30°.

Fig. 9 represents the correlation of the discharge coefficient at a side weir angle of 90° for actual and theoretical values calculated in the present work and the comparison these values with previous works, the figure shows that the majority of the data points which fall in the error do not exceed ±5%.

6. Conclusion

The present work deals with studying an alternative concept of elementary discharge coefficient along the side weir at different angles of side weir with respect to the side channel wall in the flow direction (inclined to the left). A numerical solution of two ordinary differential equations is proposed for discharge and flow depth using Euler's method. The results show that the similarity of discharge computed experimentally and theoretically, so the discharge decreased along the longitudinal side weir while the water depth increased and both values increased when the side weir angle increased with the maximum flow discharge value occurring at a side weir angle of 30° showing a 70% increase than that at 90° angle.

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