Stochastic analysis
of the quiet Sun magnetic field evolution

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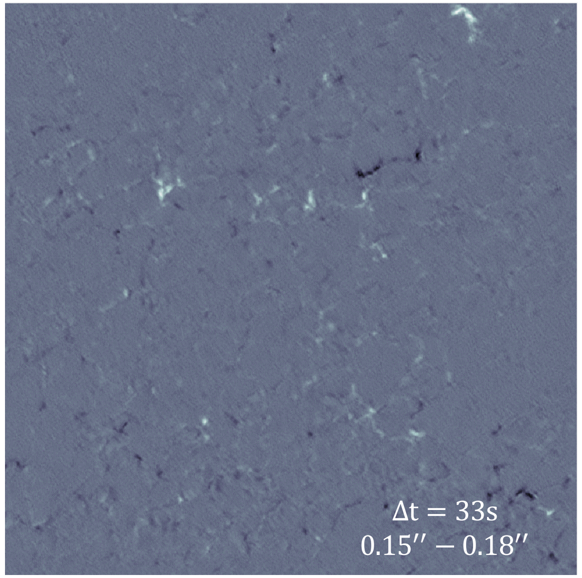
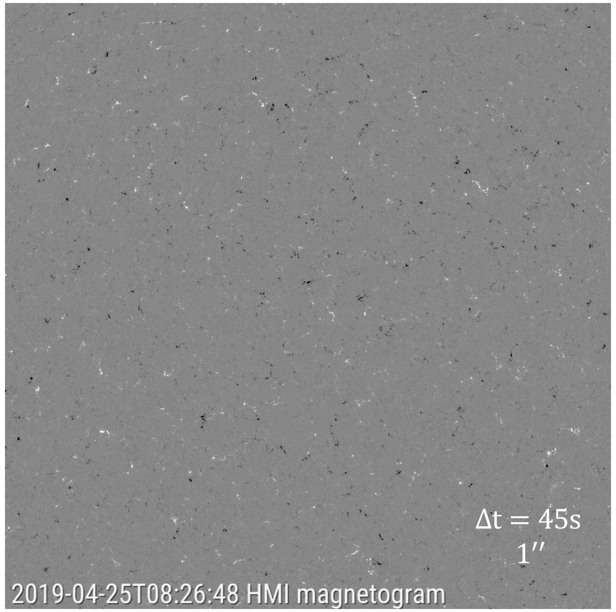
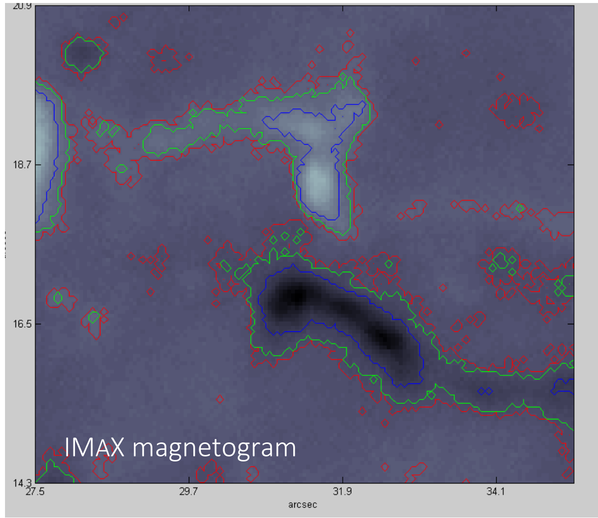
context







magnetochemistry events

1. Appearance
2. Asymmetric emergence
3. Balanced emergence
4. Balanced cancellation
5. Calving
6. Cancellation
7. Coalescence
8. Demise
9. Disappearance
10. Dispersal
11. Emergence
12. Fragmentation
13. Merging
14. Splitting
15. Unbalanced emergence
16. Unbalanced cancellation

DeForest, C. E., et al. ***2007****, ApJ, 666*

feature tracking criticism

Limited by

1. the **spatial** and **temporal** resolution of the instrumentation.

2. the difficulty of following visual **features** that **do not behave exactly like discrete physical objects**.

3. **subjectivity:**
   "… *even automated methods of solar feature tracking, produced by different authors with the intention of reproducing others' results, have myriad built-in assumptions and subjectivity of their own …****"***

Lamb D.A., et al. ***2013,*** *ApJ, 774*

the aim

to develop

an alternative characterization of evolution

excluding

uncertainties of feature tracking

overview

convert magnetic features into temporal *fluctuations*

assume statistical homogeneity of the m. field in space

apply methods of *thermodynamics* and *stochastic processes theory*

we get

new fundamental interpretations and quantitative estimates

exact relations for stochastic entropy production

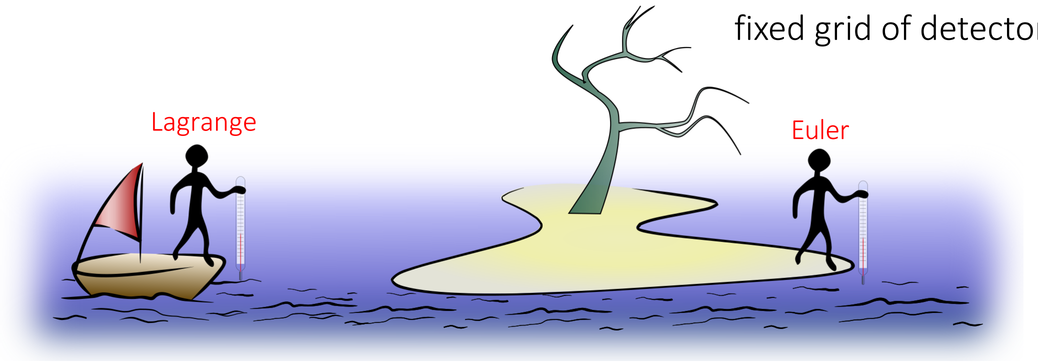
observable non-equilibrium behavior

evolutionary complexity measure

Markov property of fluctuations

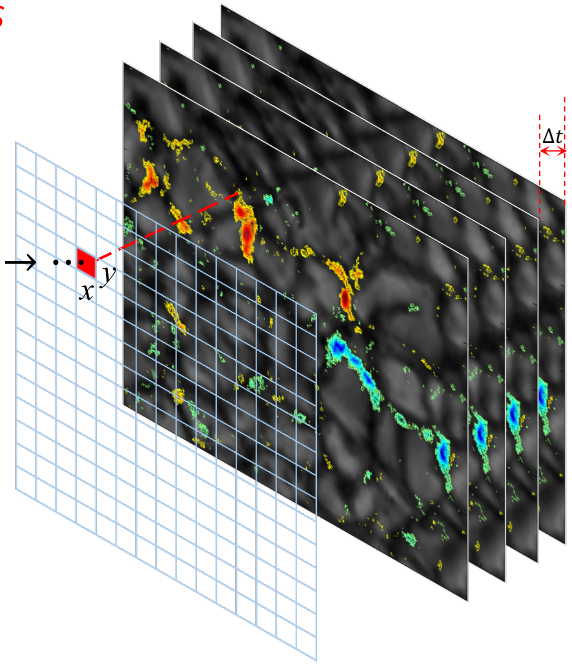
convert magnetic features into temporal *fluctuations*

fixed grid of detectors



tracking parcel of the fluid

$$b_t^{xy} \rightarrow b_{t+\Delta t}^{xy} \rightarrow b_{t+2\Delta t}^{xy} \rightarrow \cdots$$



Euler r.f. & magnetograms

assume stat. homogeneity of the m. field in space

$$b_t^{xy} \rightarrow b_{t+\Delta t}^{xy} \rightarrow b_{t+2\Delta t}^{xy} \rightarrow \cdots$$

$$b_t \rightarrow b_{t+\Delta t} \rightarrow b_{t+2\Delta t} \rightarrow \cdots$$

ensemble of trajectories collected at all pixels uniformly apply theory of stochastic processes

$$Prob\ (b_t, b_{t+\Delta t}, b_{t+2\Delta t}, \dots, b_{t+N\Delta t}) = ?$$

for Markov chains:

$$p(b_t, b_{t+\Delta t}, b_{t+2\Delta t}, \dots, b_{t+N\Delta t}) = p(b_t) p(b_{t+\Delta t} | b_t) \cdots p(b_{t+N\Delta t} | b_{t+(N-1)\Delta t})$$

$N\ is\ finite$

we found Markov property in the data

Gorobets A.Y., Borrero J.M., Berdyugina S., ***2016****, ApJL, 825, L18*

observable non-equilibrium behavior

$$p(b_t, b_{t+\Delta t}, b_{t+2\Delta t}, \dots, b_{t+N\Delta t}) = p(b_t)p(b_{t+\Delta t}|b_t)\cdots p(b_{t+N\Delta t}|b_{t+(N-1)\Delta t})$$

$$p(b_{t+\Delta t} = j | b_t = i) = [\mathbb{W}_{ij}] = \begin{pmatrix} p_{11} & \cdots & p_{s1} \\ \vdots & \ddots & \vdots \\ p_{1s} & \cdots & p_{ss} \end{pmatrix}$$

transition matrix

for **ergodic** Markov chains:

$$p(b_{t+n\Delta t}|b_t) = [\mathbb{W}_{ij}]^n = \begin{pmatrix} p_{11} & \cdots & p_{s1} \\ \vdots & \ddots & \vdots \\ p_{1s} & \cdots & p_{ss} \end{pmatrix}^n$$

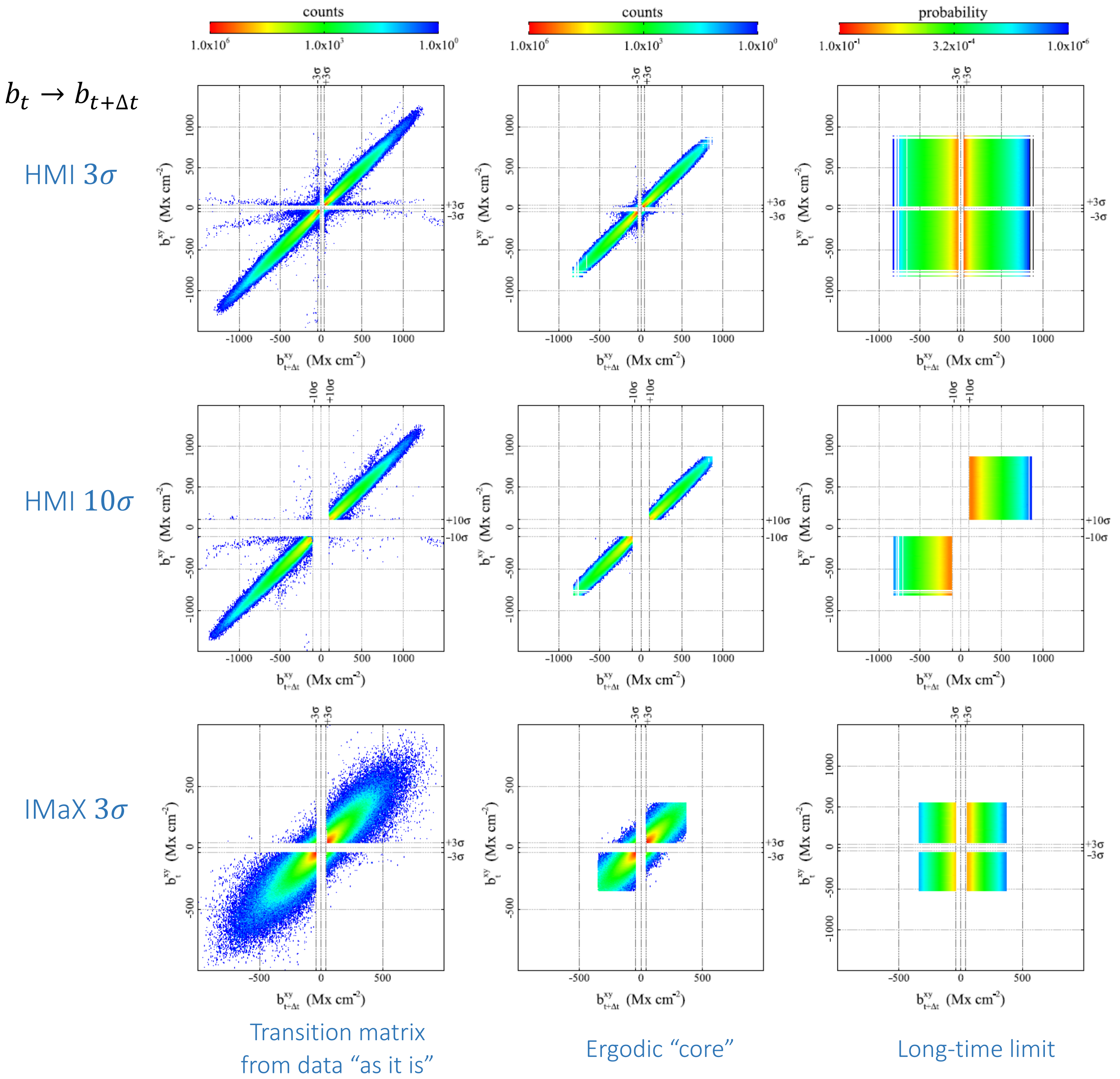
arbitrary step c.p.d.f.

$$\pi(b_n) = \lim_{n\to\infty} p(b_n|b_i)$$

stationary limit, unique *p.d.f.*

vanishing dependence on the initial state

*Gorobets, A.Y., Berdyugina, S.V., Riethmüller, T.L., et al., **2017**, ApJS, 233(1), 5*



evolutionary complexity measure

How fast QS at different resolutions becomes equilibrated

$$\pi(b_n) = \lim_{n\to\infty} p(b_n|b_i) \equiv Diag[\mathbb{W}_{ij}]^n$$

stationary limit, unique *p.d.f.*

$$\lim_{n\to\infty} \{ \sum_n \pi(b_n) \equiv Tr[\mathbb{W}_{ij}]^n \} = 1$$

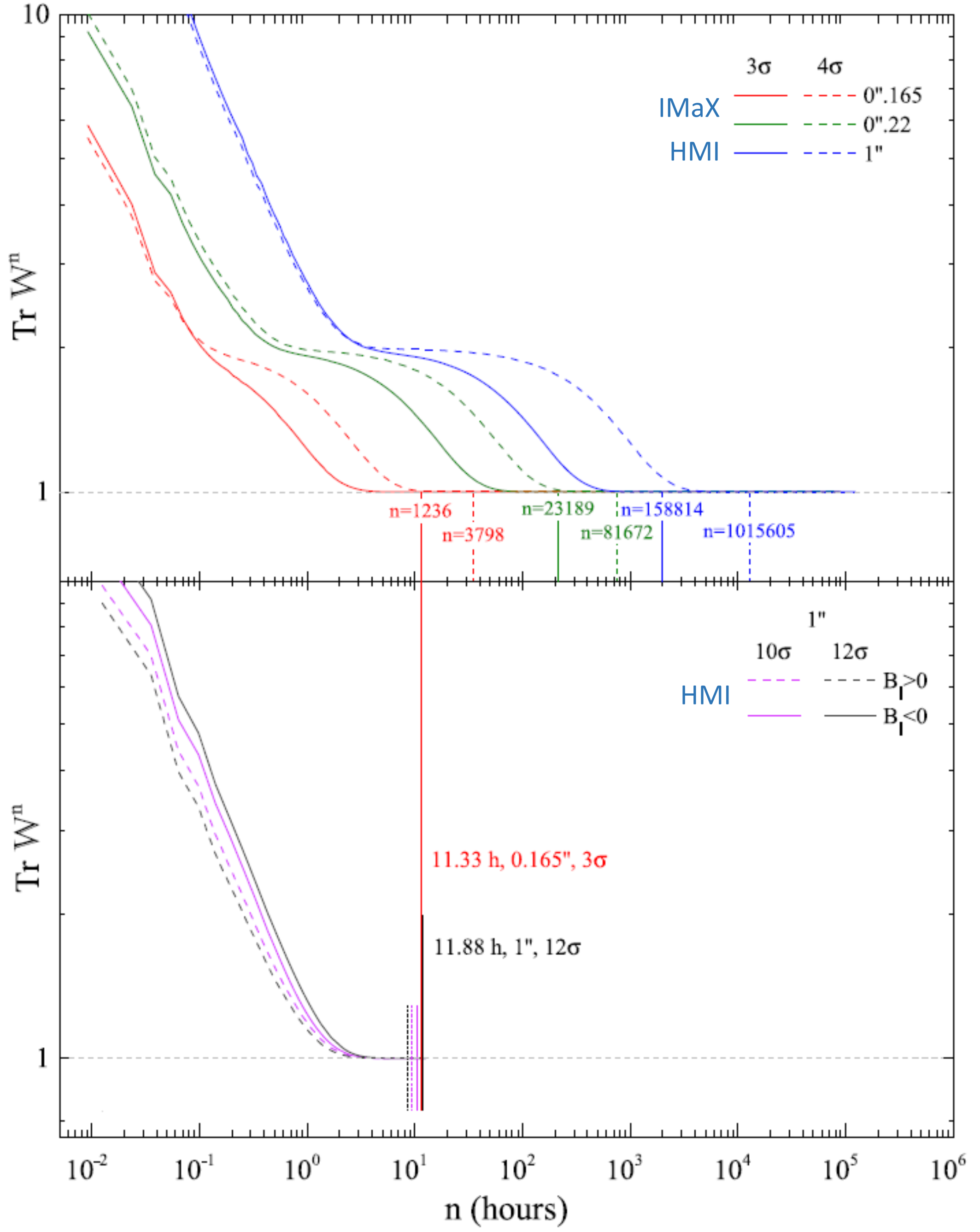
p.d.f. “self-normalization“ in the long-time limit

$t_{eq} = n : |Tr[\mathbb{W}_{ij}]^n - 1| < 10^{-7}$

equilibration time

Cutoff Level	Sunrise II/IMaX		HMI
$k\sigma$	0.″165	0.″22	1″
k	hr	hr	hr
3	11.33 *3.07*	212.57 8.9 (*days*) *3.5*	1985.18 82.7 (*days*) *6.4*
4	34.82	748.66 31.2 (*days*)	12695.1 529 (*days*)
10	–[a]	–[a]	$9.36^{B_{\rm LOS}>0}$
			$10.45^{B_{\rm LOS}<0}$
12	–[a]	–[a]	$8.44^{B_{\rm LOS}>0}$
			$11.88^{B_{\rm LOS}<0}$

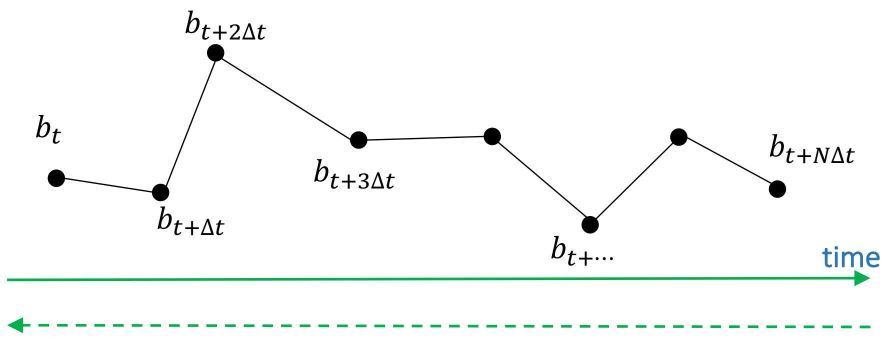
[a] Not applied due to poor statistics.



*Gorobets, A.Y., Berdyugina, S.V., Riethmüller, T.L., et al., **2017**, ApJS, 233(1), 5*

exact relations for stochastic entropy production

Microscopic violations of the Second Law



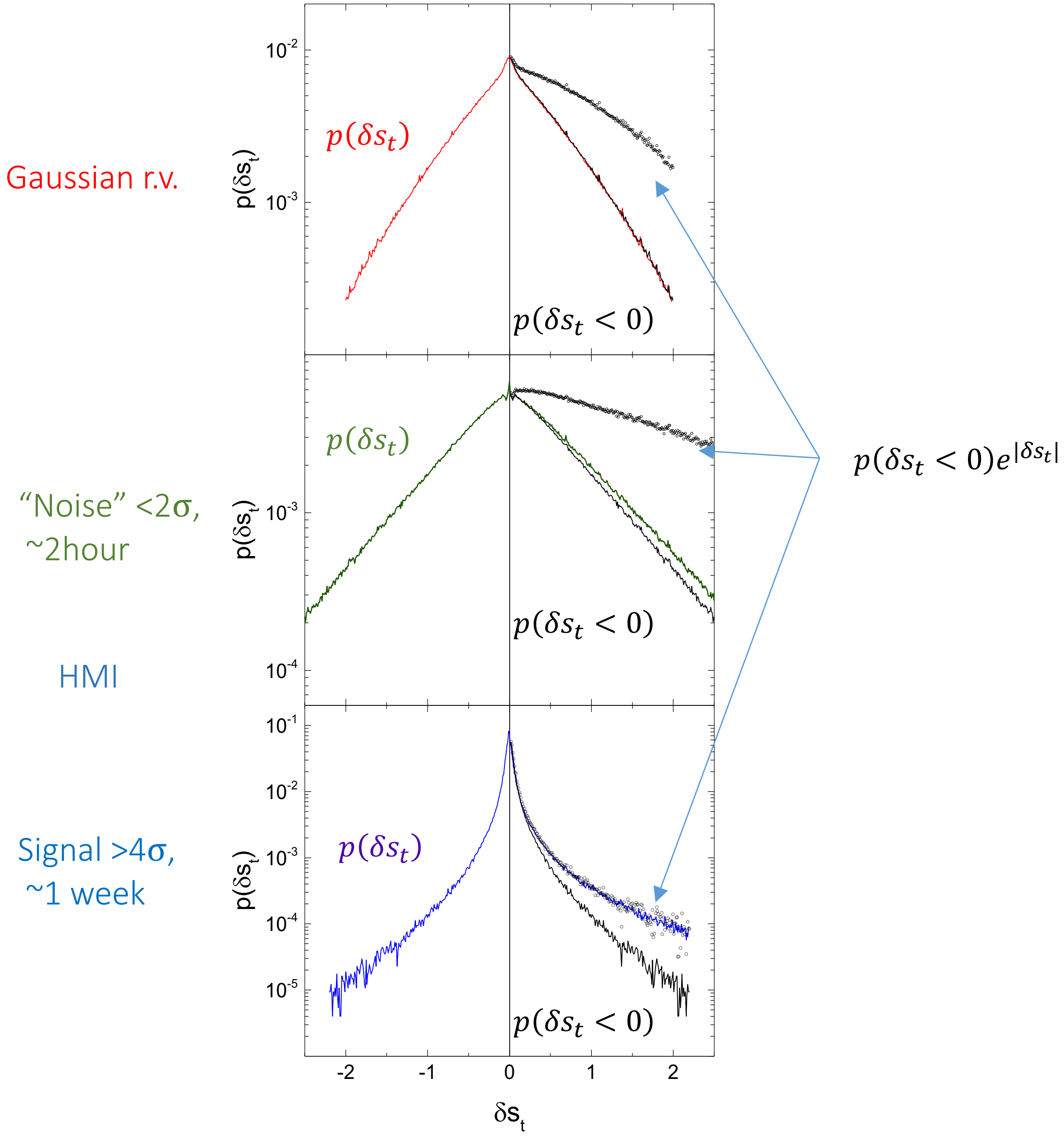
total entropy production

$$\delta s_t = \ln \frac{p(b_t \to b_{t+\Delta t} \to b_{t+2\Delta t} \to \cdots \to b_{t+N\Delta t})}{p(b_{t+N\Delta t} \to \cdots \to b_{t+2\Delta t} \to b_{t+\Delta t} \to b_t)}$$

$$\delta s_t = \begin{cases} > 0 & \text{forward is more probable = ln(>1)} \\ < 0 & \text{reverse is more probable = ln(<1)} \end{cases}$$

Fluctuation Theorem

$$\frac{p(\delta s_t > 0)}{p(\delta s_t < 0)} = \exp\{|\delta s_t|\}$$



1977: Bochkov G. N. & Kuzovlev Y. E. **Sov. Phys. JETP** 45, 125,30 (theory)
1993: Evans D. J., et al. **Phys. Rev. Lett.** 71, 2401 (simulation)
1998: Ciliberto S., Laroche C., ***J. Physique IV***, 8, 215 (first experimental test in Rayleigh–Benard convection)

*Gorobets, A.Y., Berdyugina, S.V., **2019**, MNRAS Lett., 483, L69*

summary

- There is (working) alternative to the feature tracking
- The approach allows to consider observed photospheric mag. field as a continuum media
- Insight on fundamental physics from observations
- Fluctuation Theorem is confirmed for astronomical body (*first time)*
- New tool for statistics of “energy-like variables”
- QS can be treated theoretically as the NESS thermodynamic system
- New criteria to match observations and simulations
- Rate of convergence in the long-time limit gives quantitative measure differentiating QS observations at different resolution