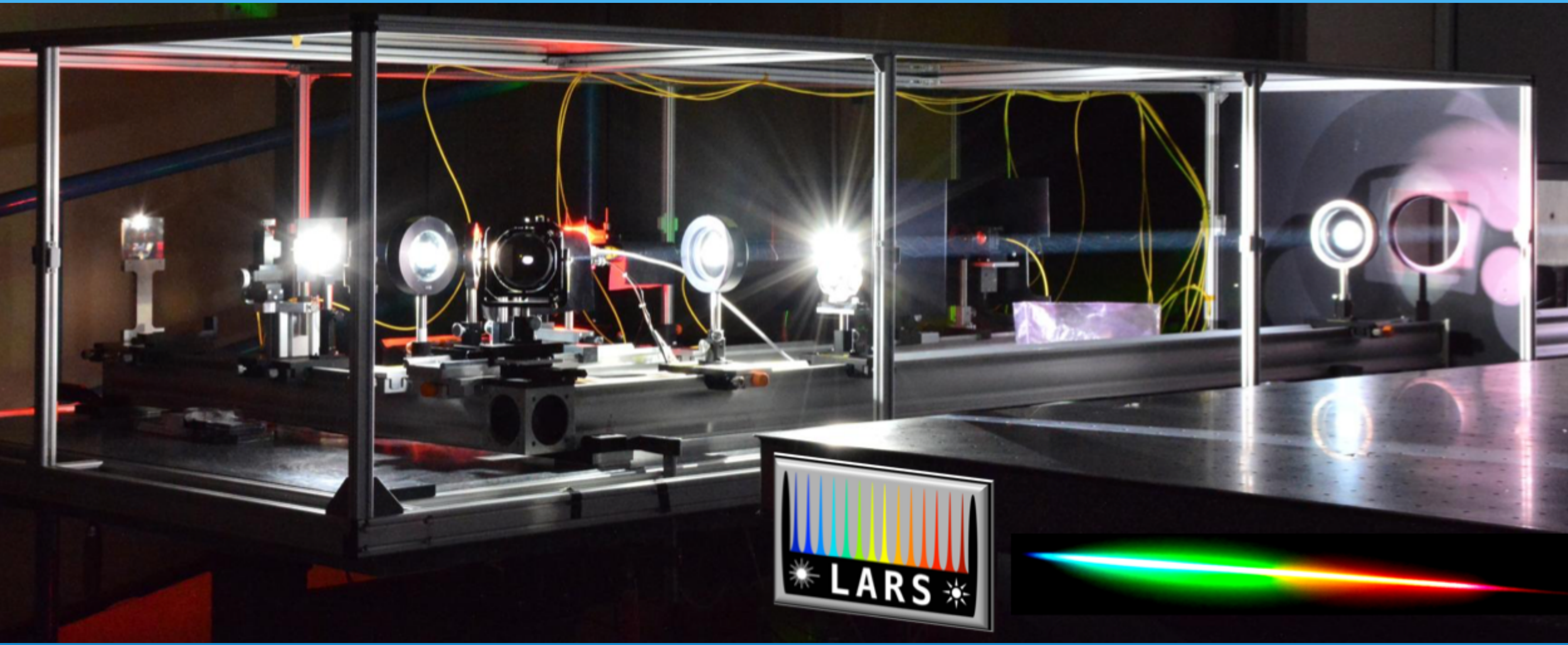
Solar spectroscopy:
Is the Sun a bright object?



A Week Above The Clouds 2019

Prof. Dr. Wolfgang Schmidt

TOC

Spectroscopy is the backbone of observational astrophysics

- *Color*
- *Temperature*
- *Chemical composition*
- *Radial velocities of stars and stellar objects*
- *Motion on stellar surfaces*
- *Oscillations*
- *Rotation*
- *Magnetic field (Zeeman effect)*
- *(Cosmological) Redshift*

Spectrographs

✳ **3 types of spectrographs:**

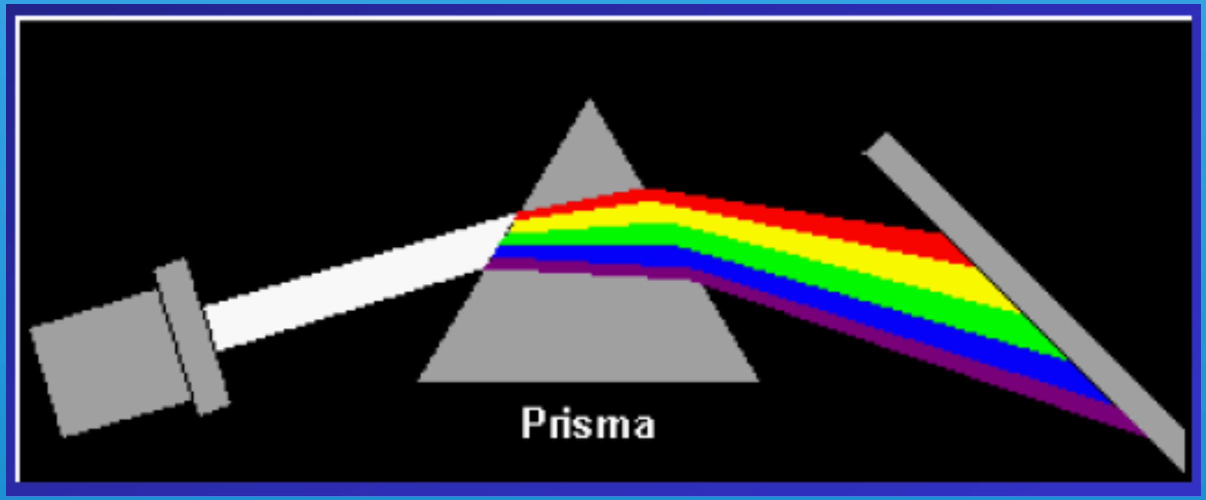
- ***Prism spectrograph***
- ***Grating spectrograph***
- ***Filter spectrograph***

✳ **Prism spectrograph:**

- Makes use of wavelength-dependent refraction:

  μ(l) = A + B/(l - C)

  A, B, C: Hartmann-Konstanten
- Dispersion increases rapidly toward the blue

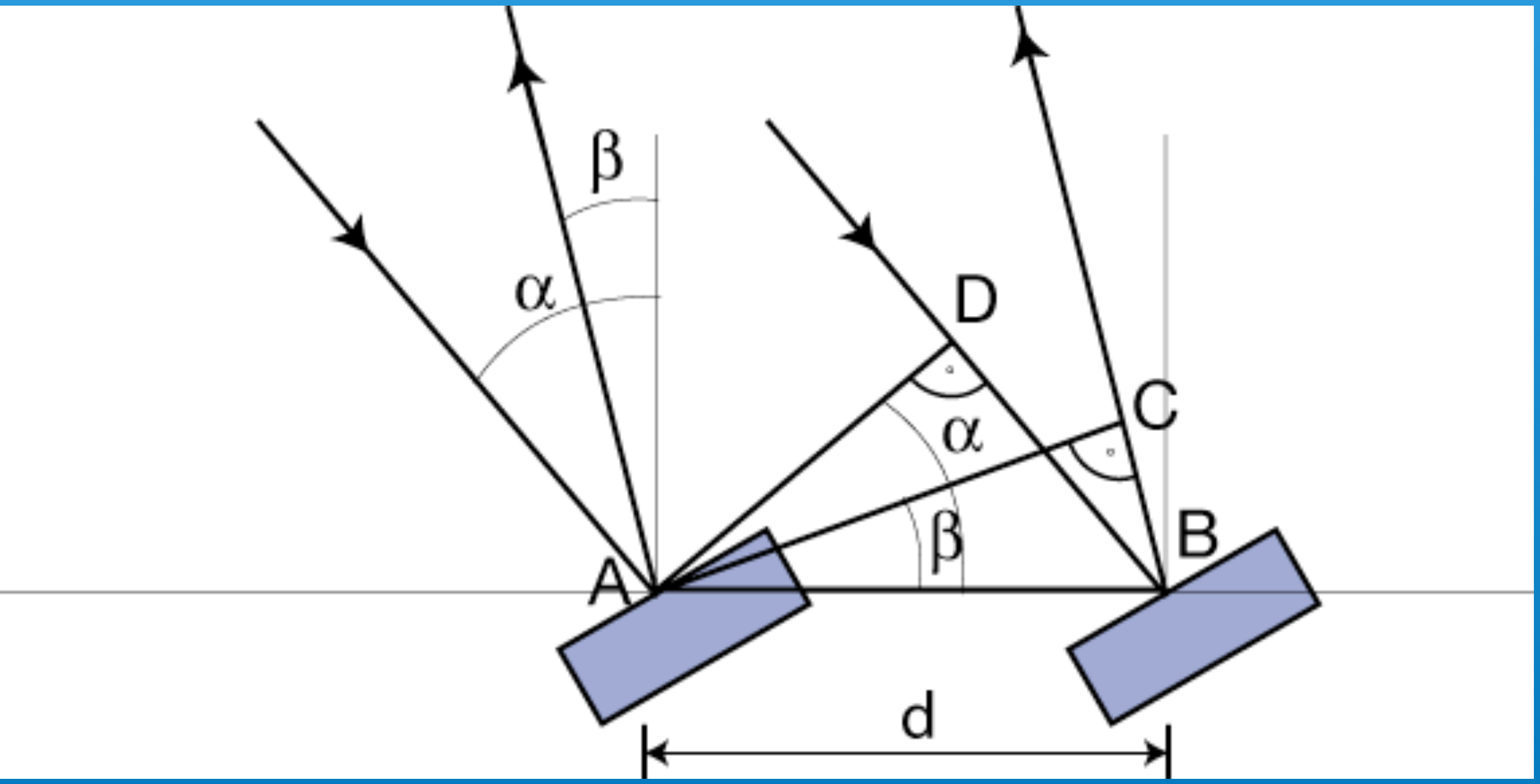


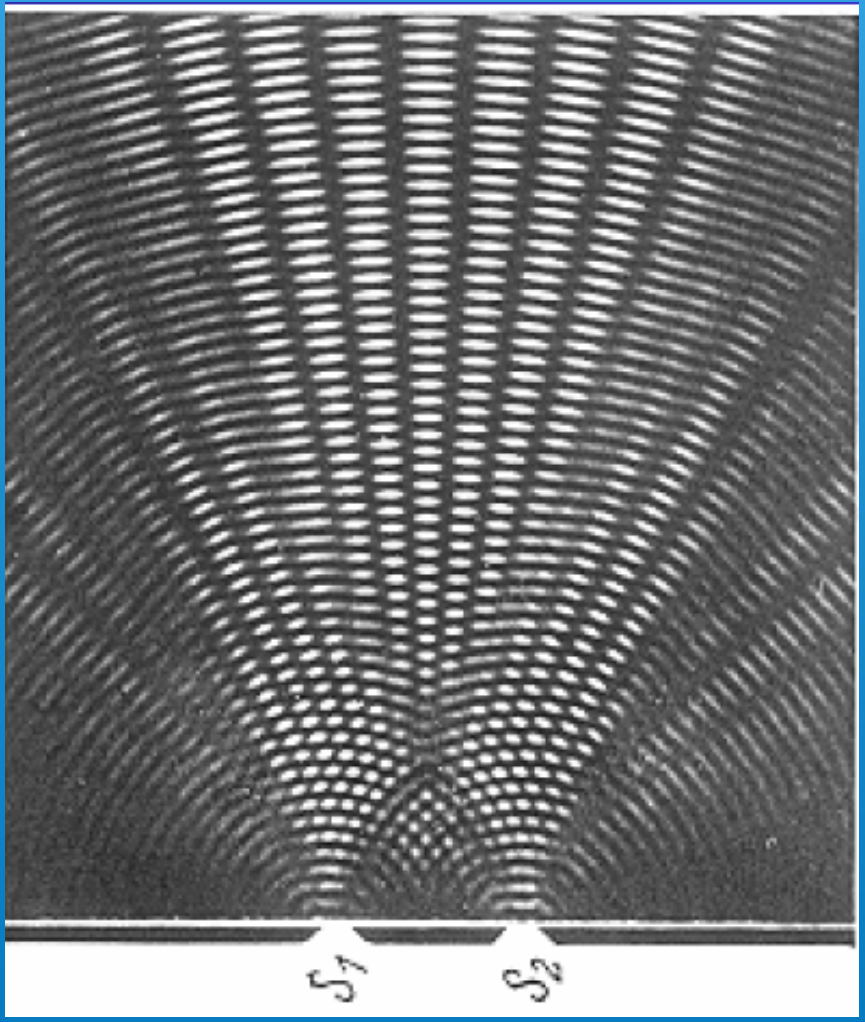
Material	**A**	**B**	**C**
Kronglas	1.477	3.20 x10-8	-2.1x10-7
Flintglas	1.603	2.08 x 10-8	1.43x10-7

$$\frac{d\beta}{d\lambda} = \frac{-AB}{\pi(\lambda - C)^2}$$

Diffraction (double slit)

Phase difference between outgoing rays	$\delta = \bar{DB} + \bar{BC} = d(\sin\alpha + \sin\beta)$
Constructive interference	$d(\sin\alpha + \sin\beta) = m \cdot \lambda$





Grating spectrograph

Grating equation:

$$m \cdot \lambda = g(\sin\alpha + \sin\beta)$$

Angular dispersion:
(1st derivative of grating equation)

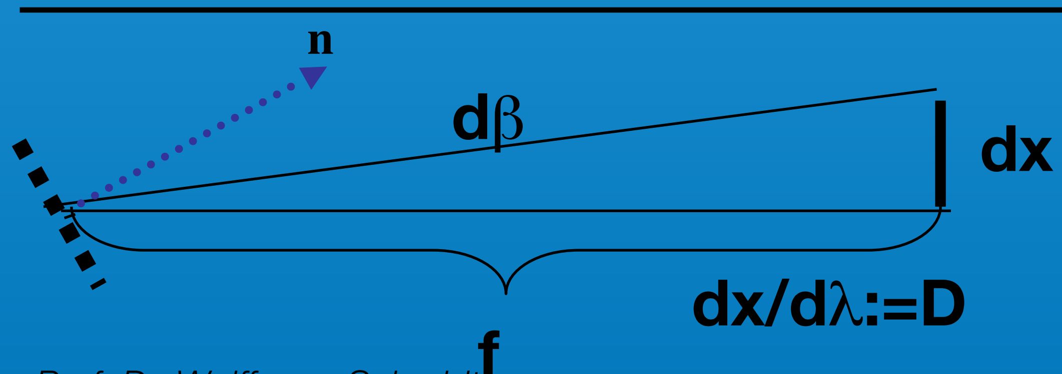
$$d\alpha \cos\alpha + d\beta \cos\beta = \frac{m}{g} d\lambda$$

$$\alpha = const. \rightarrow d\alpha = 0$$

$$\frac{d\beta}{d\lambda} = \frac{m}{g}\frac{1}{\cos\beta} = \frac{\left(\frac{\sin\alpha}{\cos\beta} + \tan\beta\right)}{\lambda}$$

$$\frac{d\beta}{d\lambda} = \frac{2 \cdot \tan\beta}{\lambda}$$

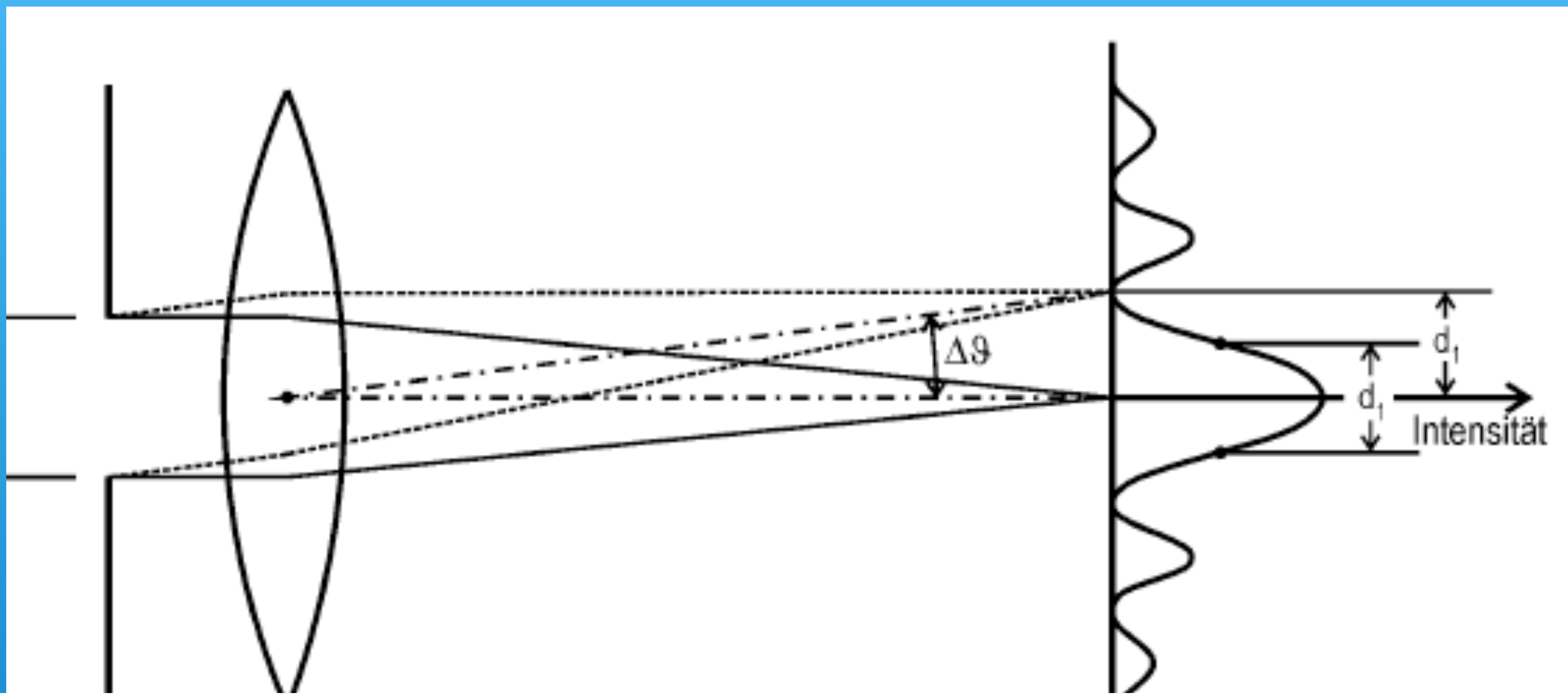
$$(\alpha = \beta)$$

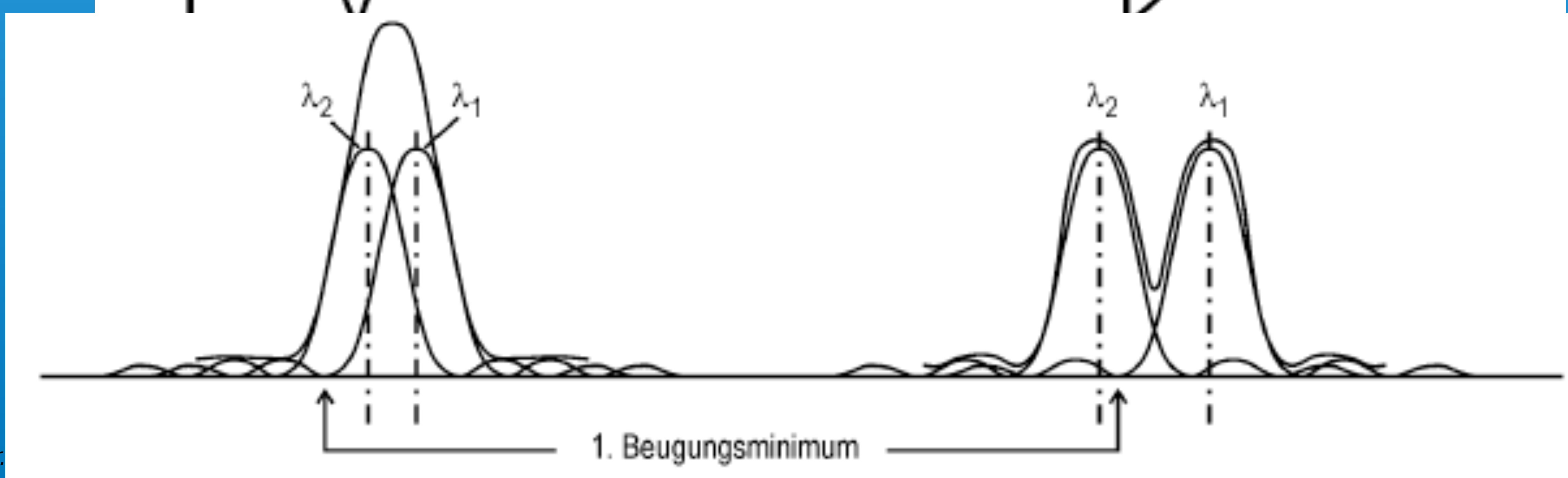


Linear dispersion (α=β): $D = f \cdot \frac{2\tan\beta}{\lambda}$

Resolving power

s





Resolving Power II

- The resolving power of a grating does NOT depend on the grating constant, and does NOT depend on the diffraction order.
- The well-known equation **R=mN** is easily derived from the previous equations.
- The resulting resolving power is a theoretical value for a slit width of ZERO, i.e., for vanishing intensity.
- The real width of the entrance slit reduces the resolving power (through diffraction at the edges).
- There is an optimum slit width *„Förderliche Spaltbreite“, s_0)*, for which the diffraction pattern of the slit (D_0) is equal to the slit width.

$$m \cdot \lambda = 2g \sin \beta$$

$$\rightarrow \frac{2 \sin \beta}{\lambda} = \frac{m}{g}$$

$$D_0 = f \cdot \frac{\lambda}{s_0}$$

A slit width below s_0 would only reduce the light level, but not increase the resolving power.

$$N := \frac{B}{g} \; (number\ of\ grooves)$$

$$\rightarrow R := \frac{2B \sin \beta}{\lambda} = \frac{mNg}{g} = m \cdot N$$

Resolving power III

Relation between slit width and effective resolving power

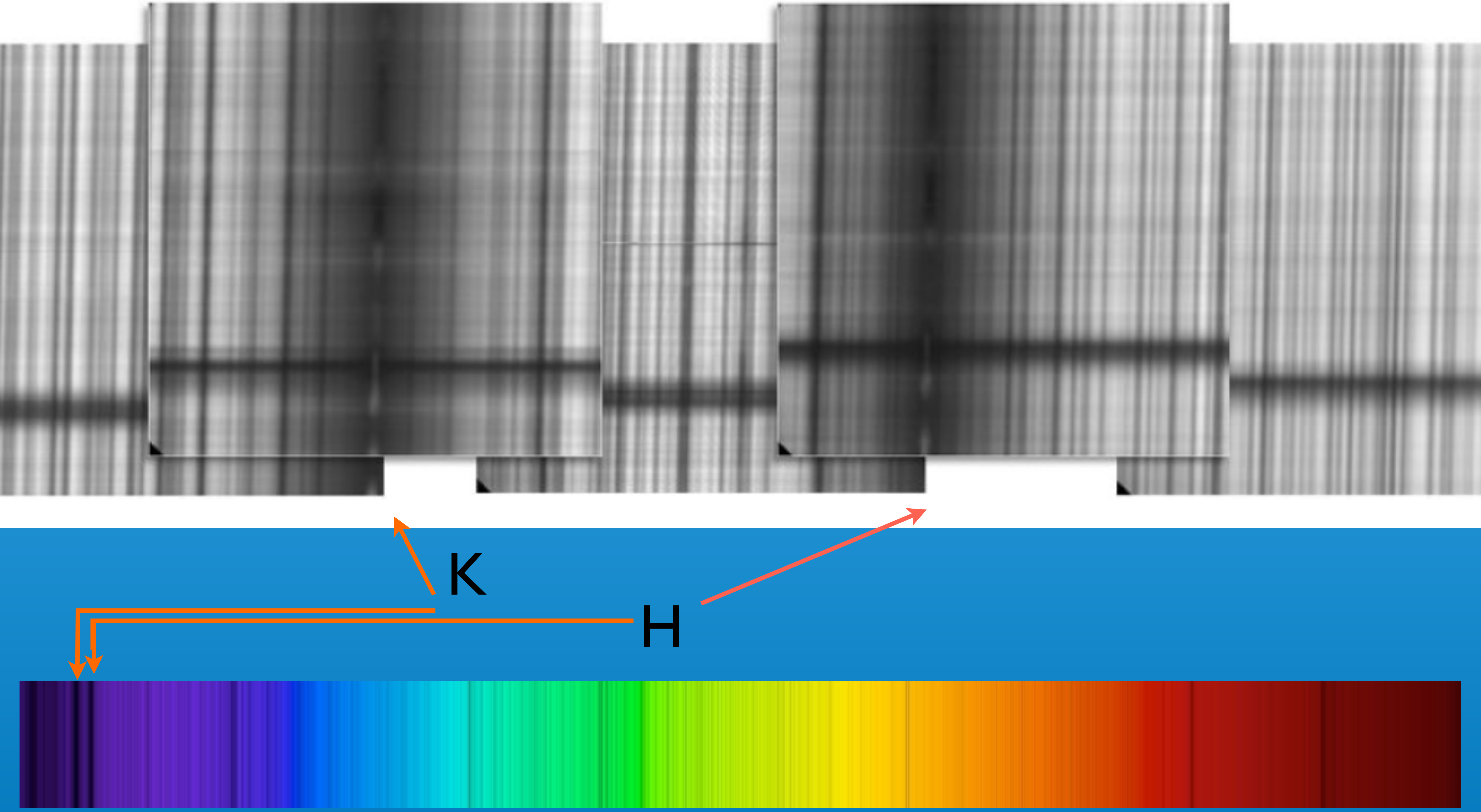
Slit width [s/s_0]	Intensity [I/I_0]	Effective resolution [%]
0.5	0.5	99.7
1.0	1.0	98.6
2.0	1.9	94.3
3.0	2.6	87.2

Ingredients for high spectral resolution:

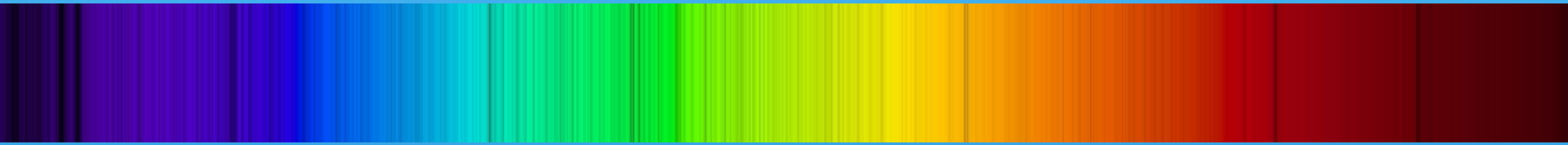
- **a large grating (large value of B)**
- **the optimum slit width**
- **large diffraction angles (high angular dispersion)**
- **suitable focal length (to match the sampling of the detector)**

Ca H & Ca K

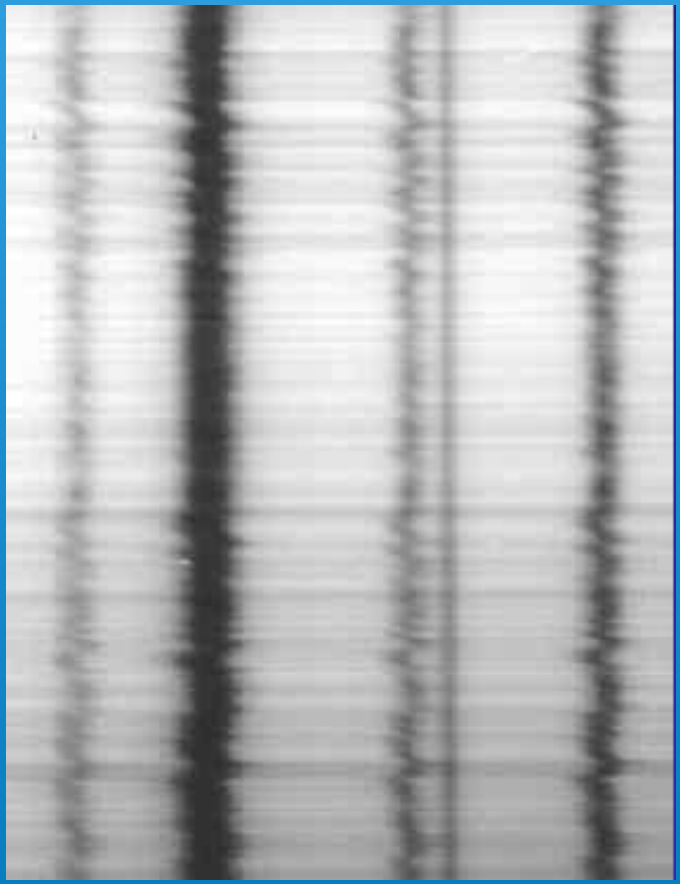
Spectrograph:
- 1st diffraction order
- 1.96 Å/mm (30 Å on chip)
- 13.5 arcsec/mm

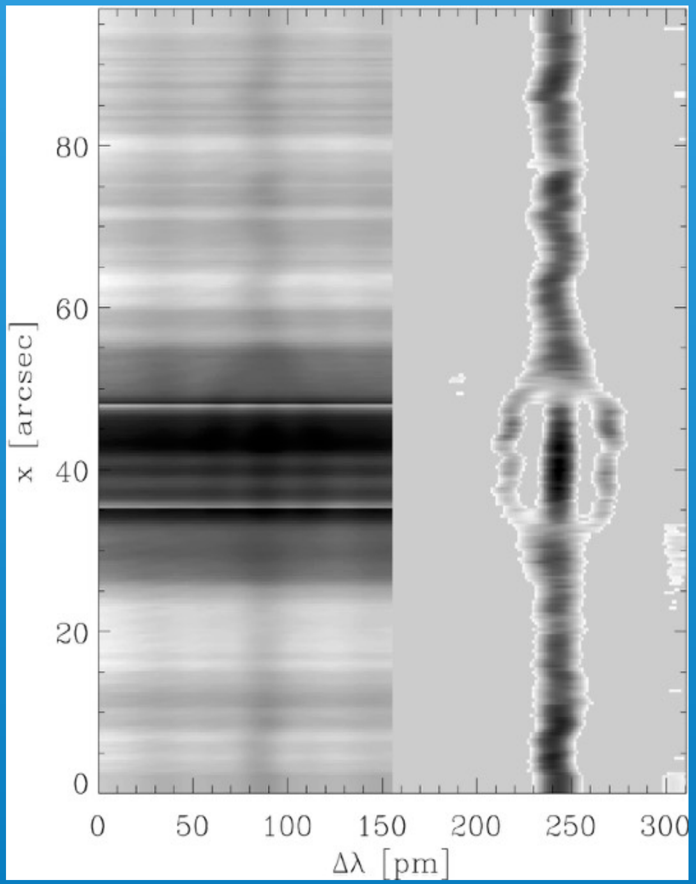


Solar spectrum



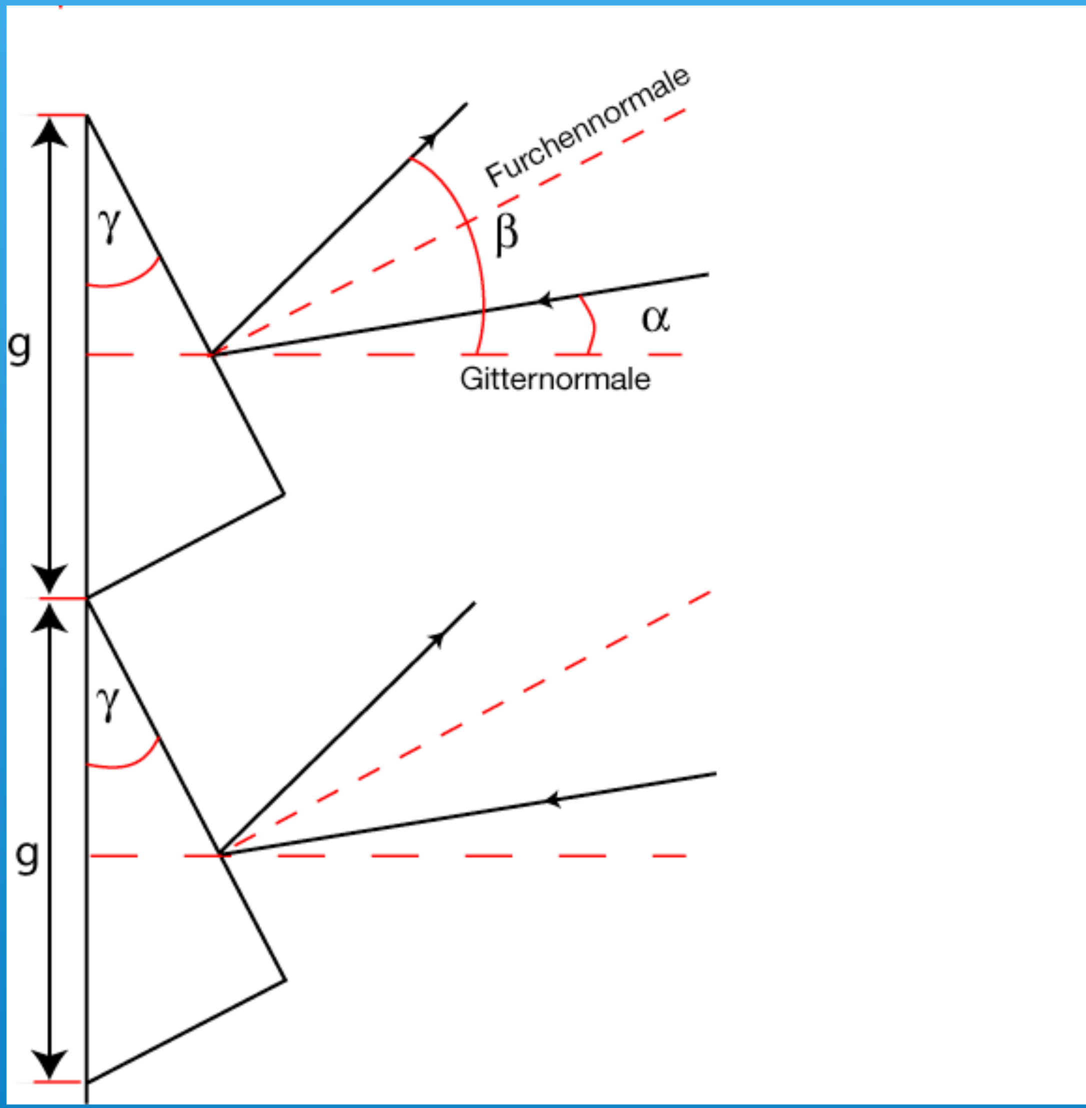
Ca II H-beta Mg Na H-alpha



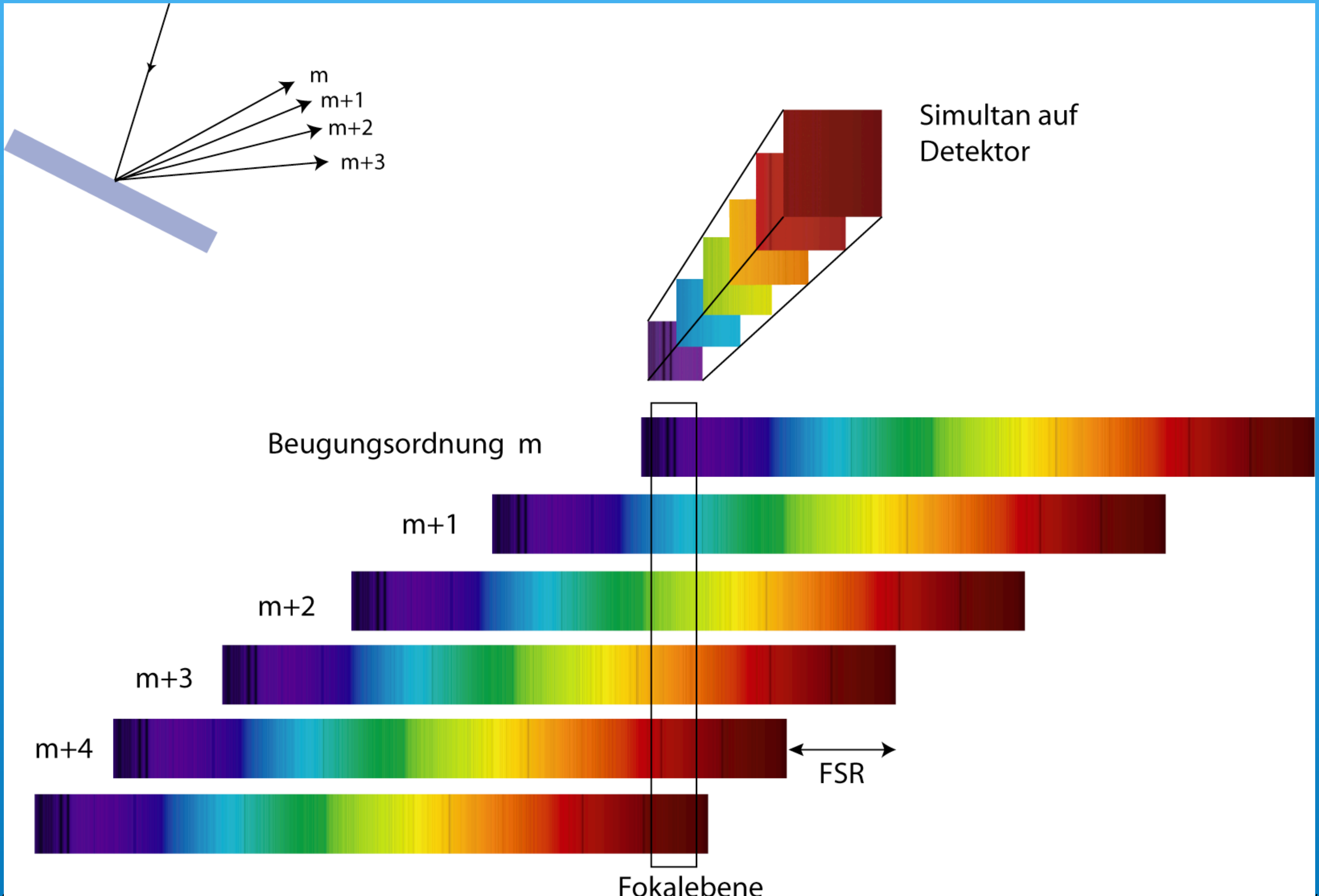


Echelle grating

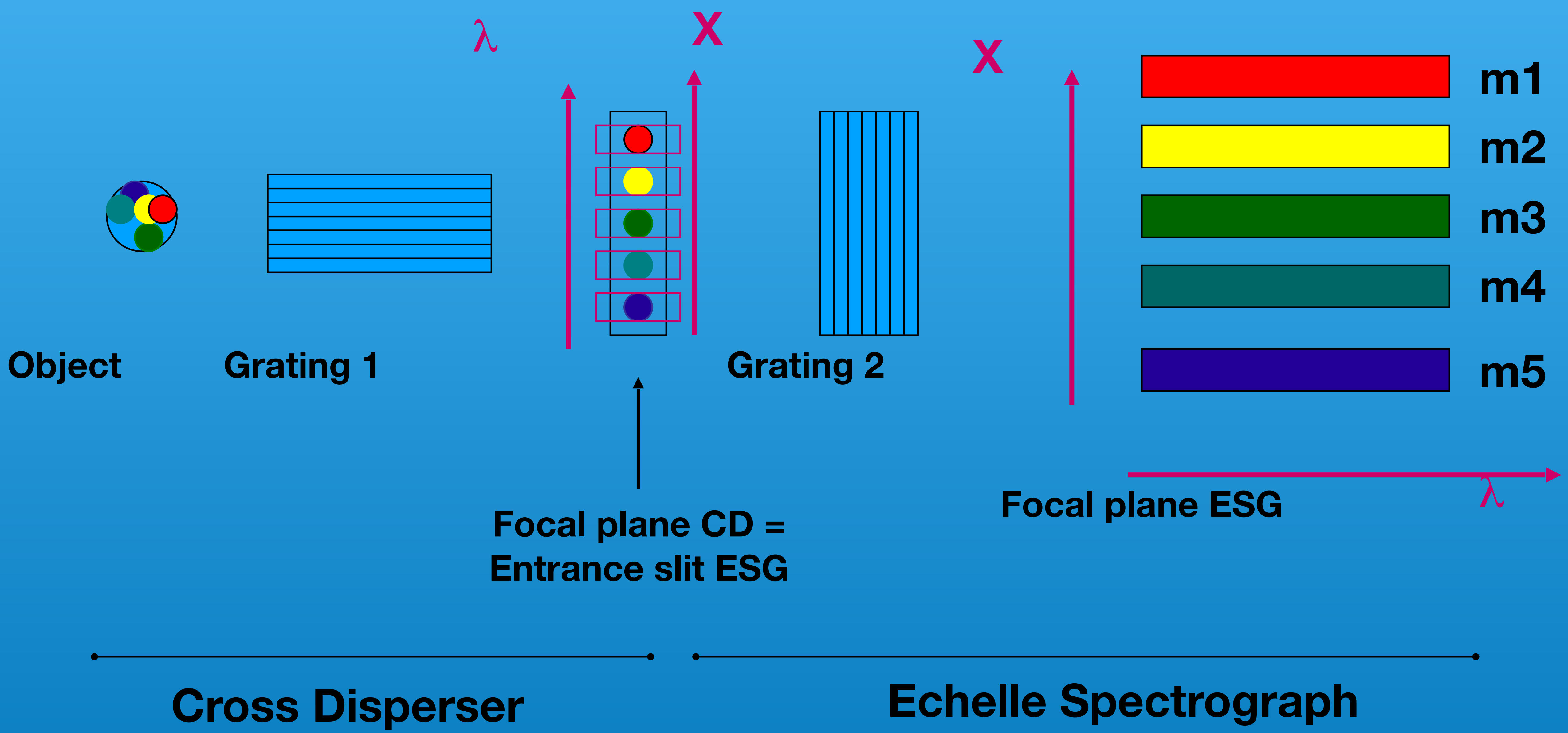
- Grating constant g
- Angle of incidence α
- Diffraction angle β
- Blaze angle γ
- g = 79 or 158 mm^{-1}
- typical Blaze: 63.4^0 (tan γ = 2.0)
- High diffraction orders(λ=630 nm at m=36)



Overlapping orders

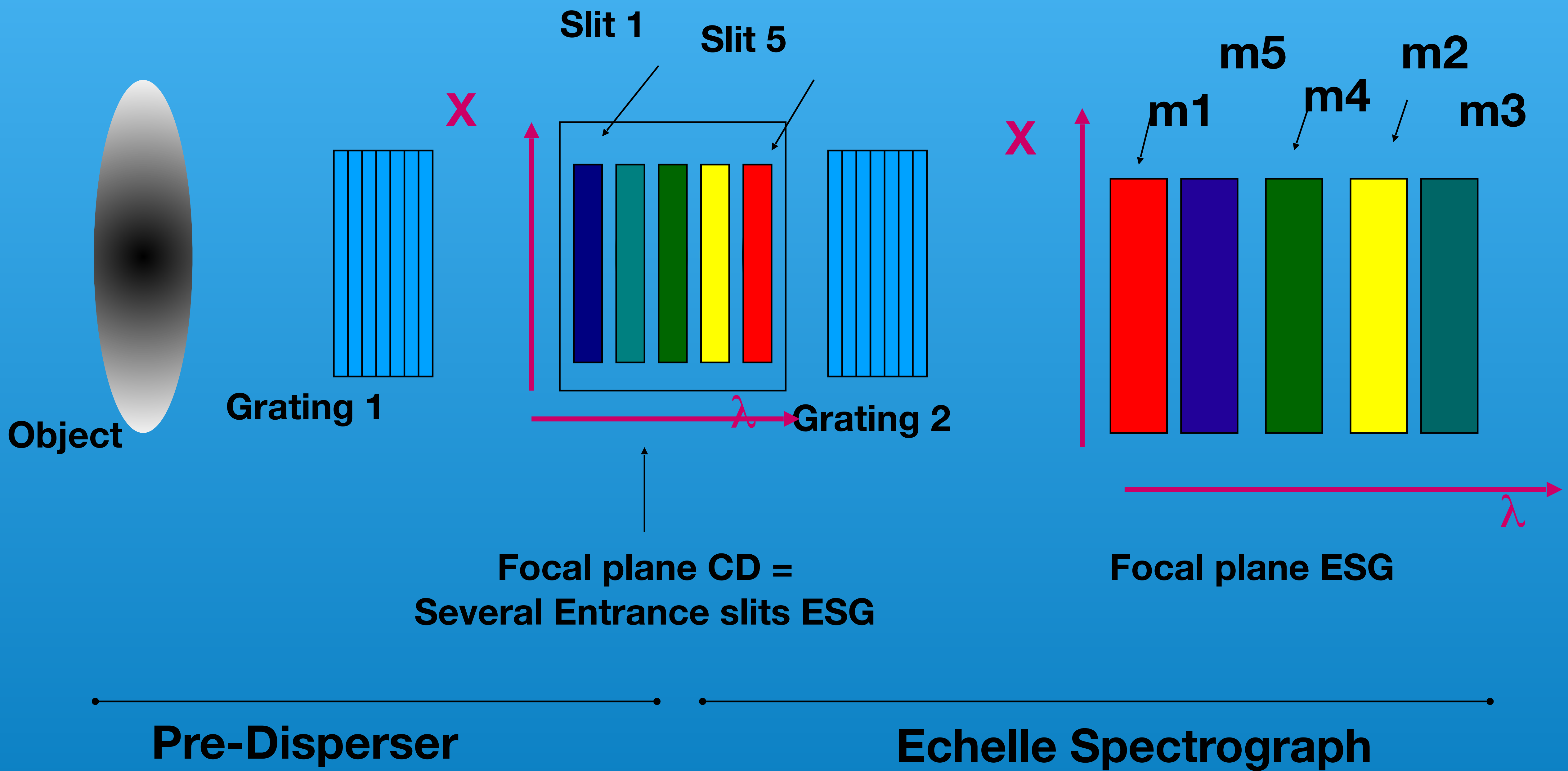


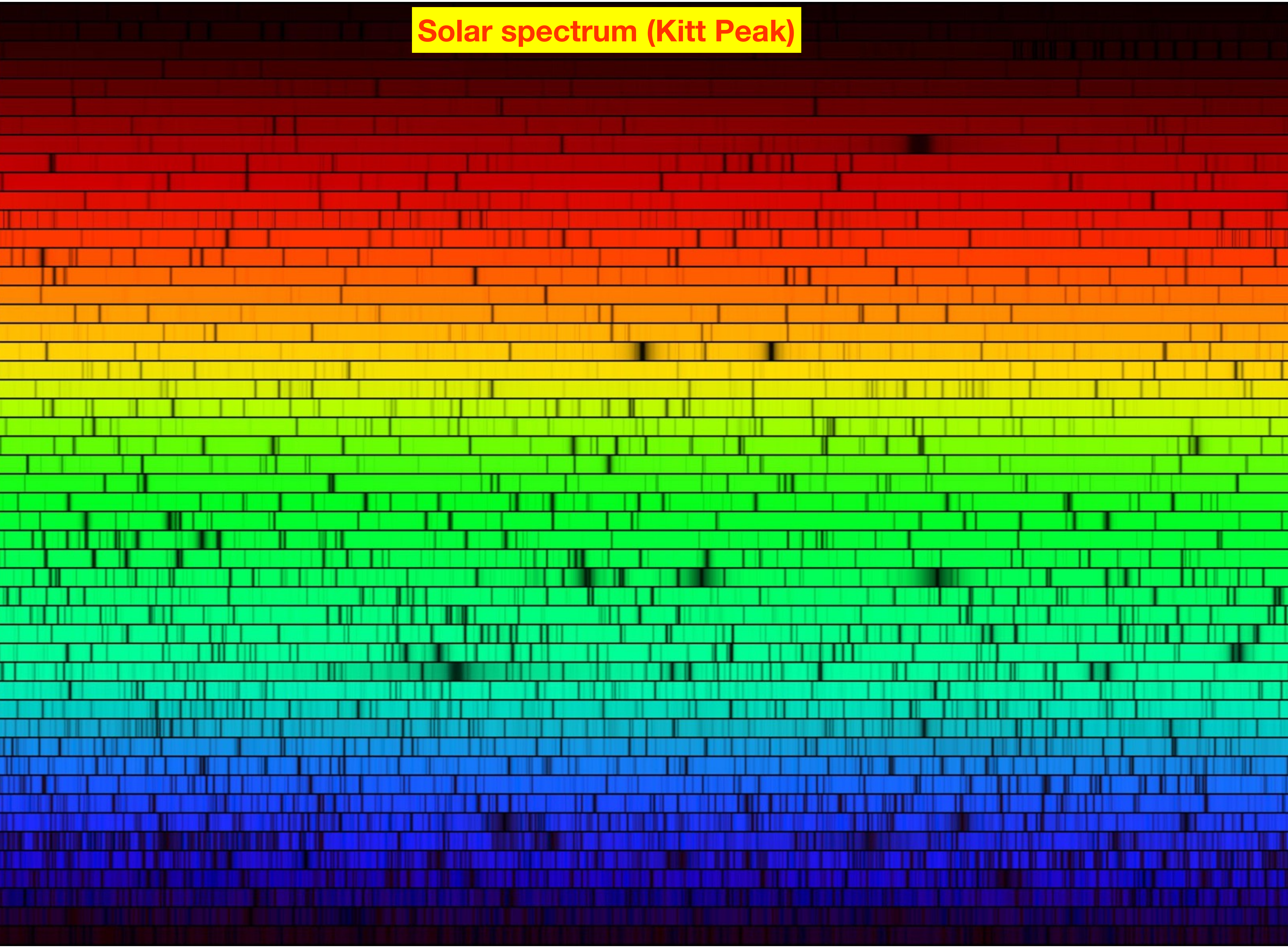
Echelle Spectrographs (II)



Echelle-Spectrographs (III)

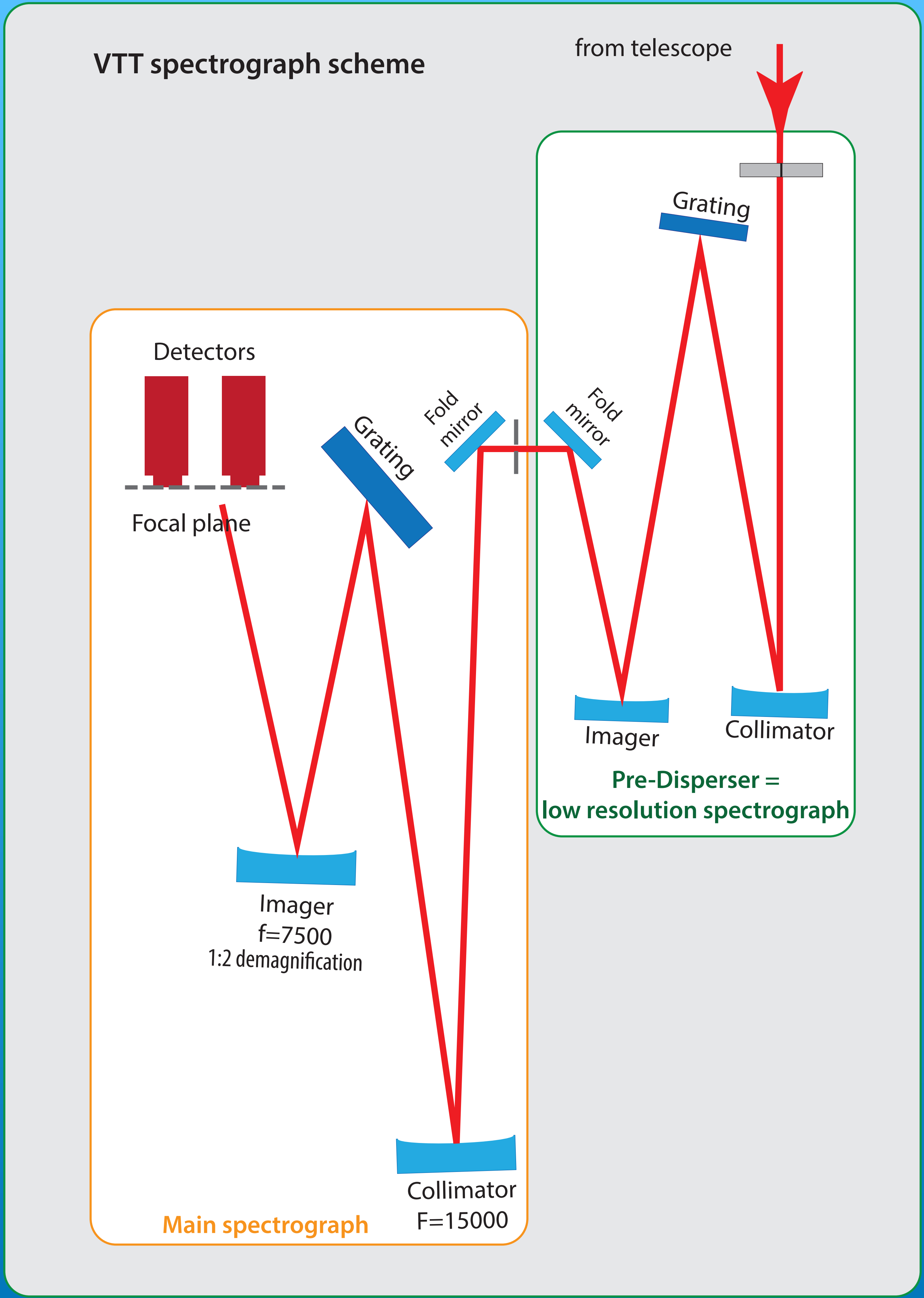
Solar Echelle-Spectrograph with predisperser



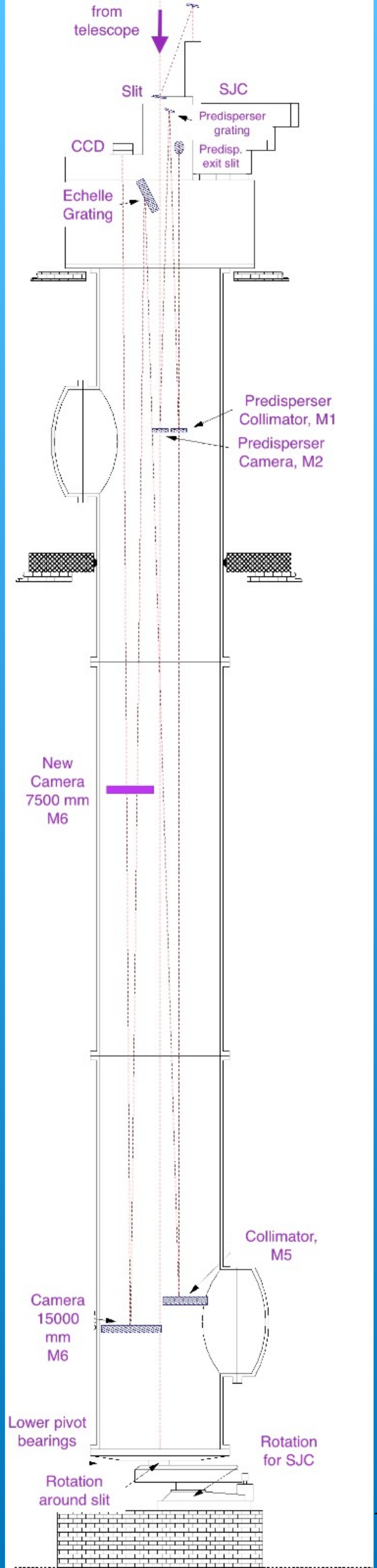

Solar spectrum (Kitt Peak)

VTT spectrograph

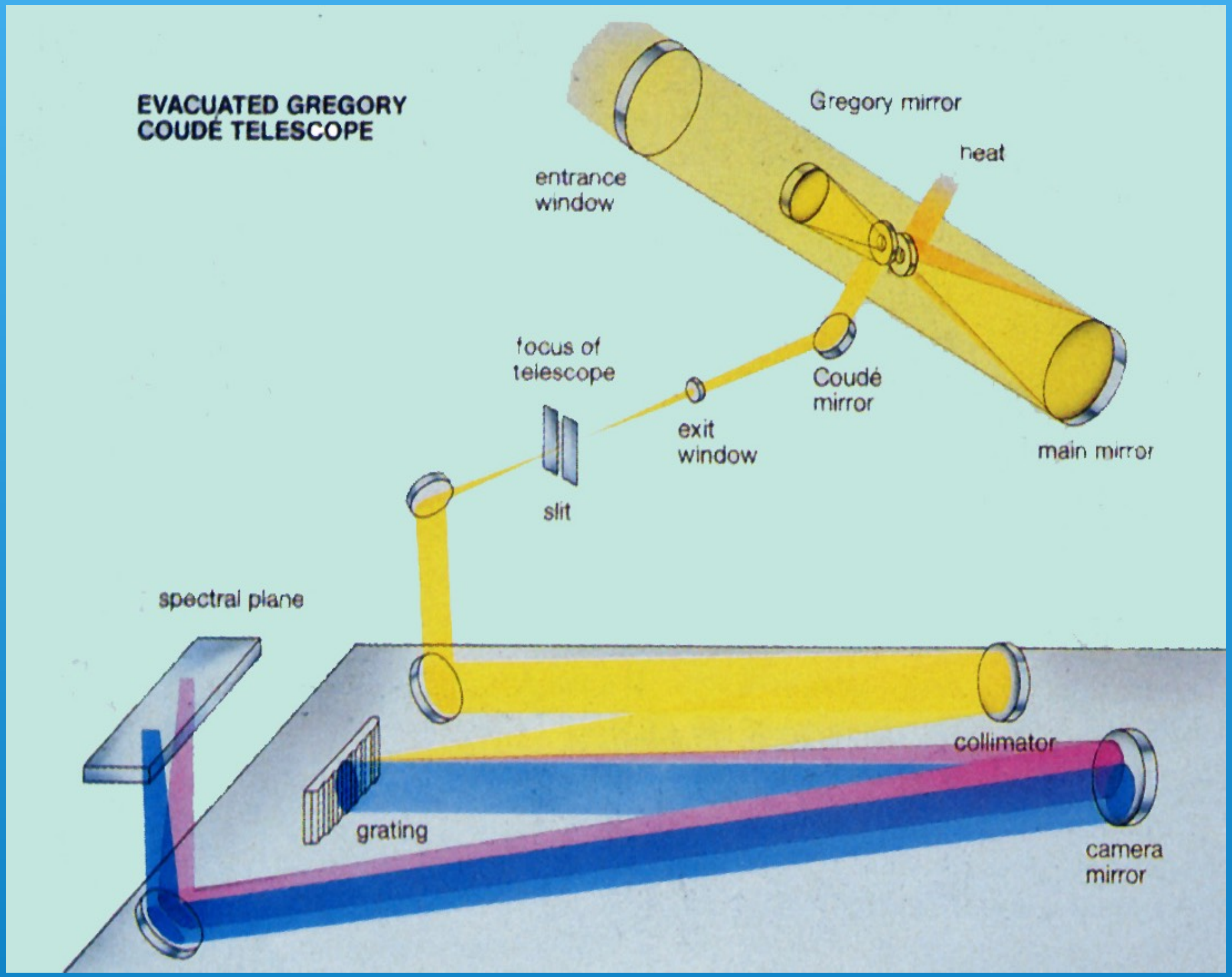
- Predisperser: low dispersion spectrum
- Exit slit of Predisperser = Entrance slit of main spectrograph: isolates one diffraction order of main spectrograph
- Main spectrograph: Echelle grating, observe 630 nm in 36th diffraction order
- Image scale:
    - 4.5”/mm at entrance slit
    - 9.0”/mm at detector
- Spectral resolution: 0.6 pm@500nm
- Linear dispersion: 20 pm/mm (1Å/cm)

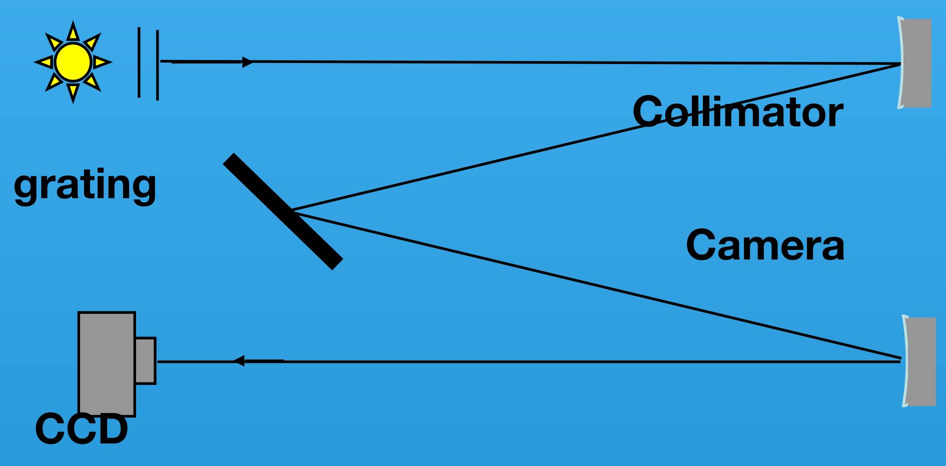


VTT Spectrograph



Czerny-Turner Spectrograph (GREGOR)





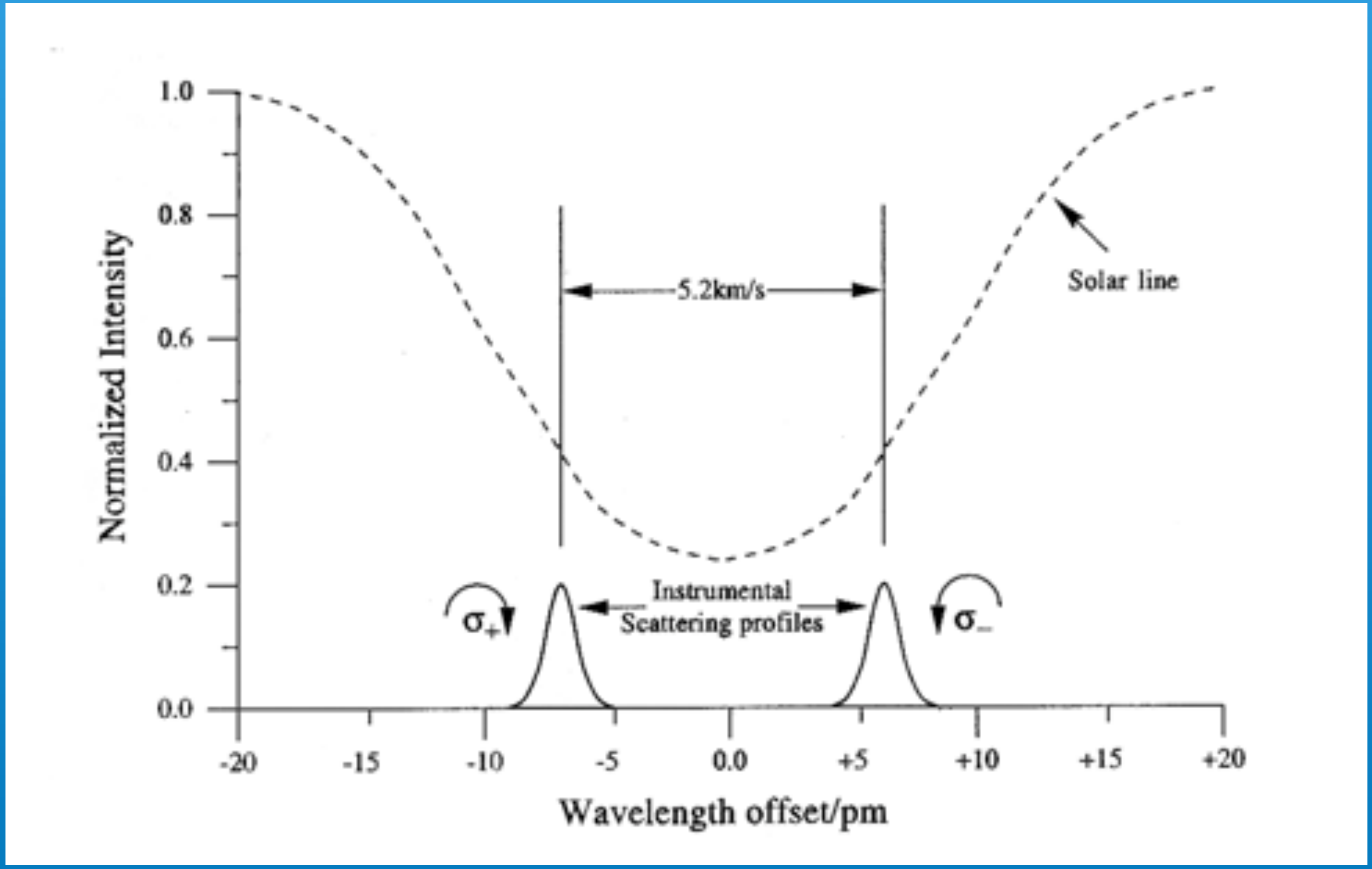
Scheme of the spectrograph of the 45cm GCT. GREGOR uses the same spectrograph layout.

Filter-Spectrometers

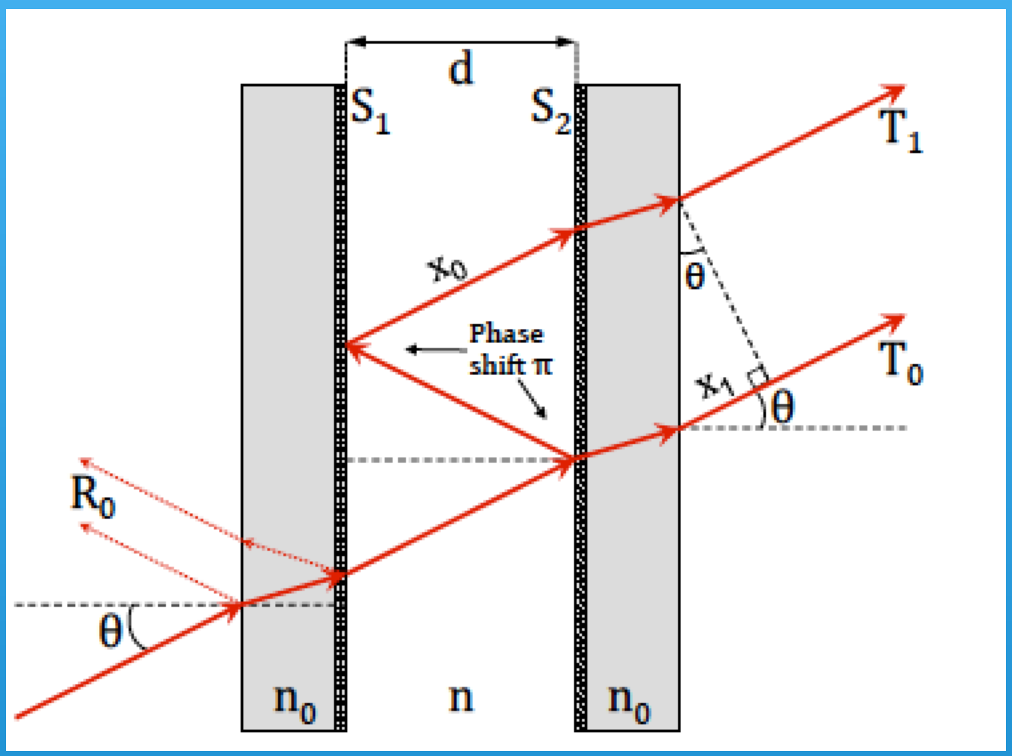
Fabry **P**érot **I**nterferometer (FPI)
Resonant **S**cattering **S**pectrometer (RSS)
Magneto-**O**ptical **F**ilter (MOF)
Michelson **I**nter**f**erometer (MIF)

RSS

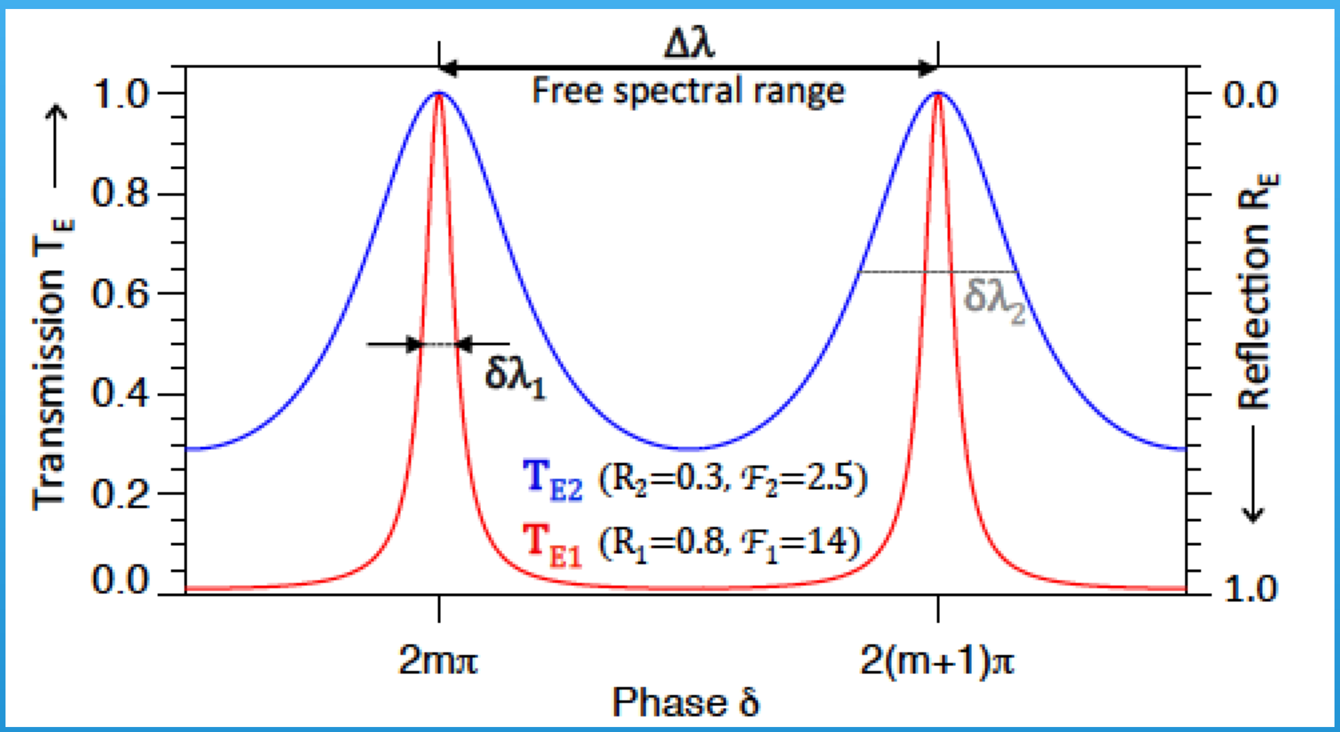
Resonant-scattering spectrometers. The resonant-scattering component within our spectrometers consists of a small silica vapor cell, containing an ensemble of potassium atoms heated to ~100 °C, so that they form a vapor. The cell is held in a strong magnetic field that moves the working parts of the vapor into the blue and red wings of the Solar Fraunhofer Line. This arrangement is shown schematically below:



Fabry-Pérot Interferometer



Light path through a Fabry-Pérot interferometer. Two transparent plates with refractive index n_0 face each other with reflecting surfaces S_1 and S_2 with plate separation **d** with index **n**. The light (red arrows) is transmitted (**T**$_i$) and reflected (**R**$_i$) under an angle of incidence θ.



Transmission of a Fabry-Pérot interferometer The transmittance functions **T**$_{E1}$ (red) and **T**$_{E2}$ (blue) for two different reflectance values **R**$_1$ = 0:8 (finesse **F**$_1$ = 14:0) and **R**$_2$ = 0:3 (**F**$_2$ = 2:5) are plotted against the phase.

Combining several FPIs

The small free spectral range is a problem for astrophysical applications: $\lambda^2/2d$.
For λ=500 nm and d=1mm we have FSR=$(5\cdot10^{-7})^2/0.002$ =1.25 nm
A grating spectrograph (m=40) has: FSR= λ/m = 12.5 nm

Using more than one FPI increases the FSR. For plate separation ratios close to 1 (Vernier-Ratios) die final resolution is given by

$$Res = \frac{\lambda}{\Delta\lambda} = \pi \frac{d}{\lambda}\sqrt{Fi(1+\varepsilon_2^2+\varepsilon_3^2)}$$

Optical configurations for FPIs

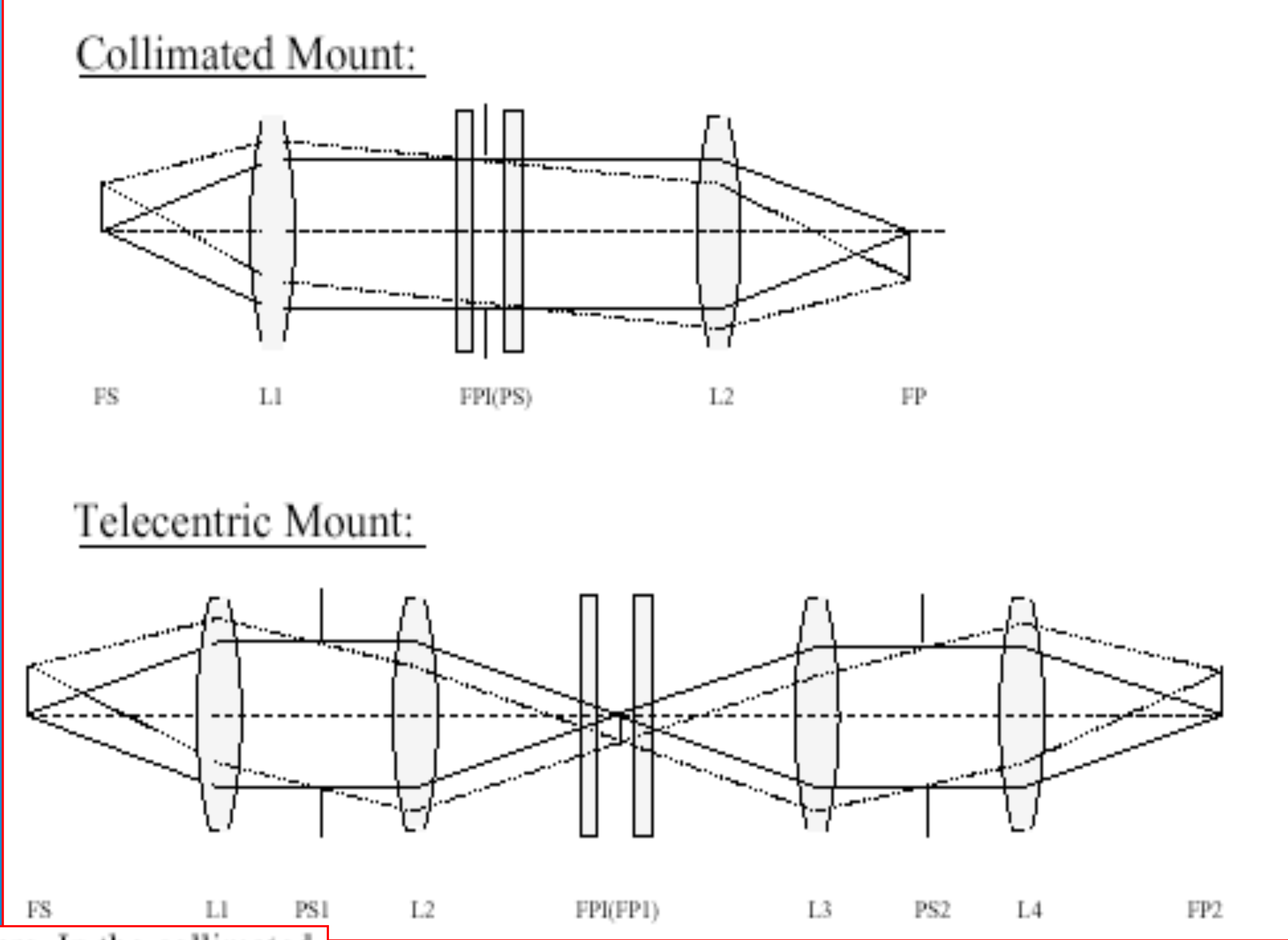
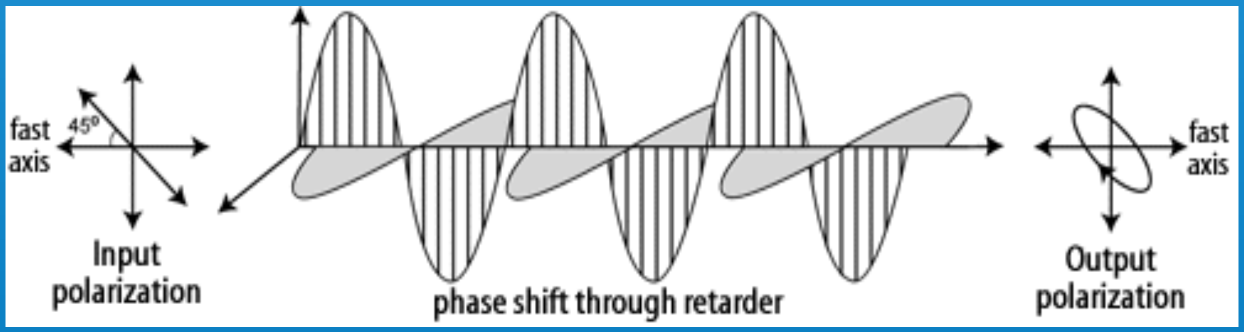
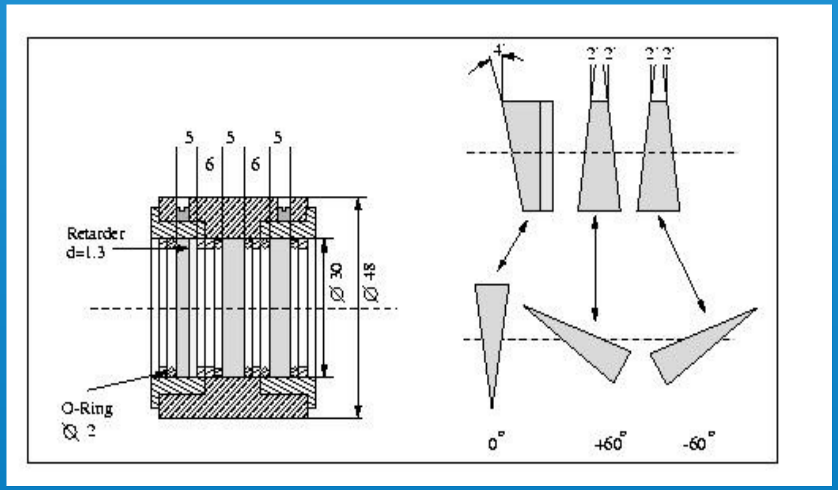


Fig. 3. Mounting concepts for FPI interferometers. In the collimated mounting, the telescope aperture (PS) is imaged into the interferometer(s), L2 is the reimaging lens. At the FPI location the beam is collimated. In the telecentric configuration, lenses L1 and L2 project the solar image (FS) into the interferometer. Lenses L3 and L4 image the solar image onto the detector (FP2).

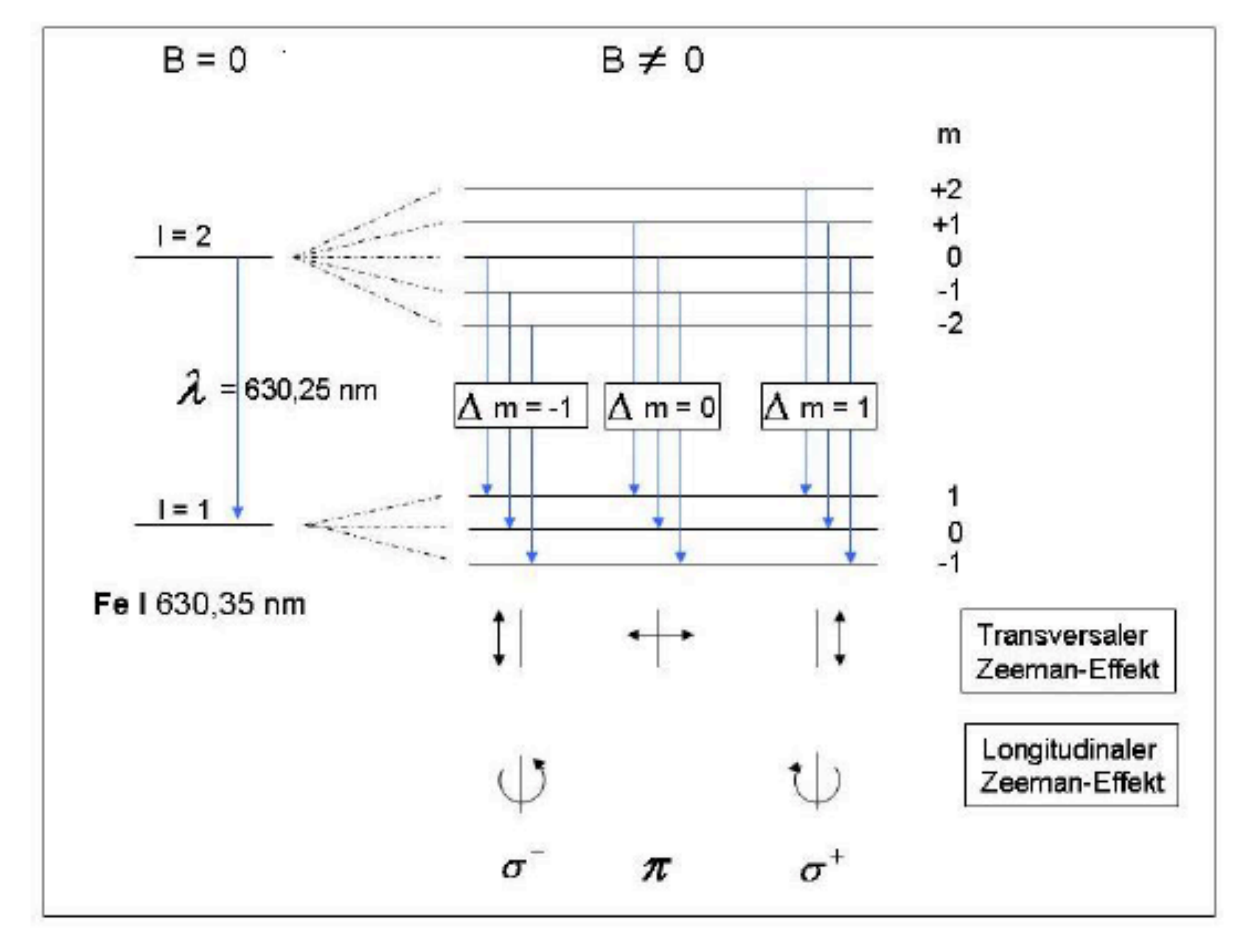
Polarimetry

- Stokes-Formalism
- Examples
- Optical components
- Polarimeter principle
- Examples: ASP, POLIS, VTF





Zeeman splitting



$$\lambda - \lambda_0 = \frac{e}{4\pi m_e} g_{eff} \lambda^2 B$$

$$B = \frac{2(\lambda - \lambda_0)}{2 \cdot 4.67 \cdot 10^{-12} \cdot g_{eff} \lambda^2}$$

History (very incomplete)

1650	Erasmus Bartholinus discovers double refraction in calcite
1690	Christian Huygens discovers extinction between crossed calcites
1808	Etienne-Louis Malus discovers polarization from reflection
1815	David Brewster discovers angle of reflection where light is totally polarized, and finds relation with index of refraction
1828	William Nicol invents the calcite polarizing prism
1844	Wilhelm Haidinger discovers the polarization sensitivity of the human eye
1852	George Stokes introduces the Stokes parameters
1858	E. Liais discovers the partial linear polarization in the solar corona
1908	George Hale finds circular and polarization in sunspots
1947	Horace Babcock discovers circular polarization in 78 Vir
1953	Karl-Otto Kiepenheuer and Horace Babcock measure magnetic fields all over the Sun

K.-O. Kiepenheuer Magnetograph

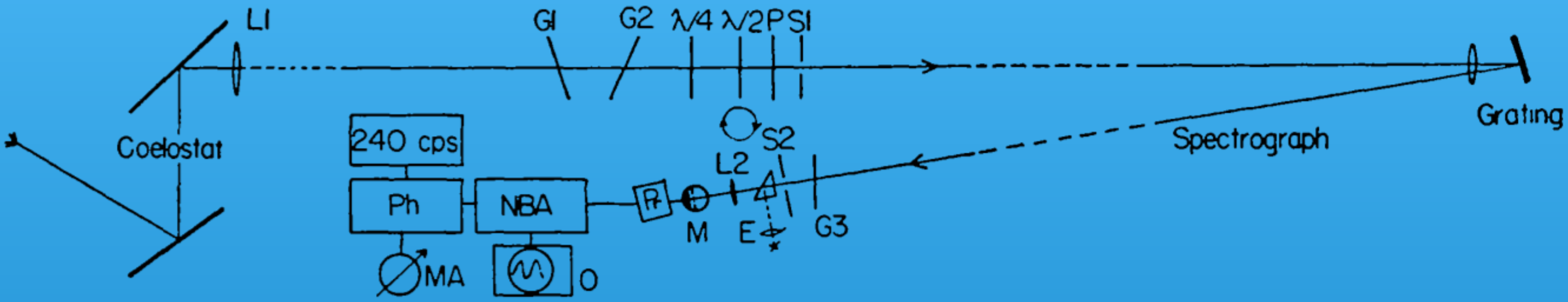


FIG. 1.—Instrumental arrangement

The construction of the instrument used in this investigation is in several details similar to that used by Thiessen.[3] As shown in Figure 1, the image of the sun is projected by the objective L_1 onto the entrance slit S_1 of a grating spectrograph. In front of the slit there are two thin plane-parallel glass plates of high quality, G_1 and G_2, which can be tilted and rotated around the tilted axis; while λ/2 is a half-wave mice plate (30 mm in diameter) which rests in the hollow shaft of a synchronous motor of special design rotating with 3600 r.p.m., λ/4 is a fixed quarter-wave[4] plate, and P is a fixed polaroid. The

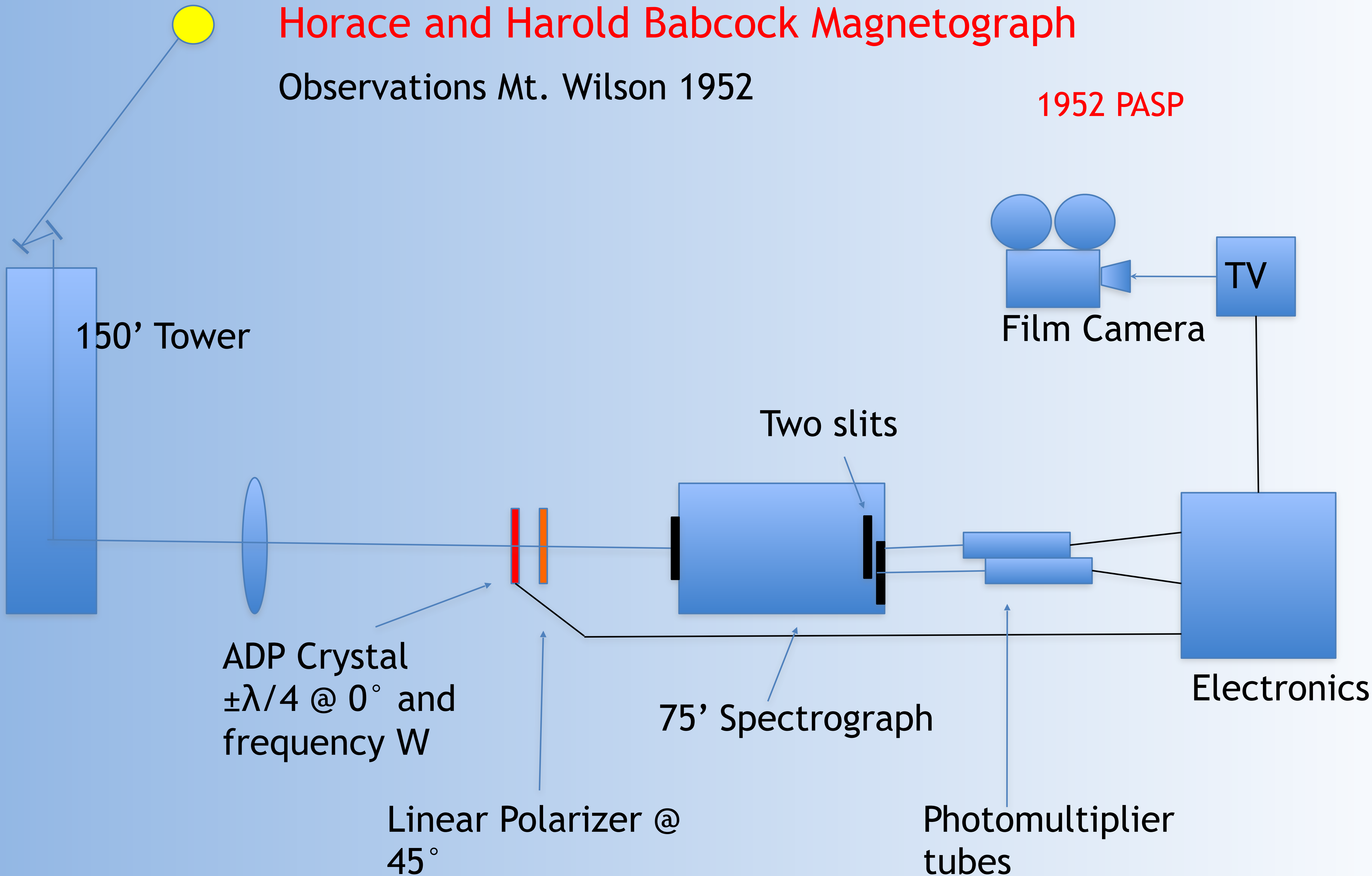
K.O.Kiepenheuer magnetograph II

Built and tested at Yerkes 1950-1951.
Observations at Mt. Wilson 1951

Horace and Harold Babcock Magnetograph

Observations Mt. Wilson 1952

1952 PASP



$Q \propto \cos(2Wt)$

$V \propto \sin(Wt)$

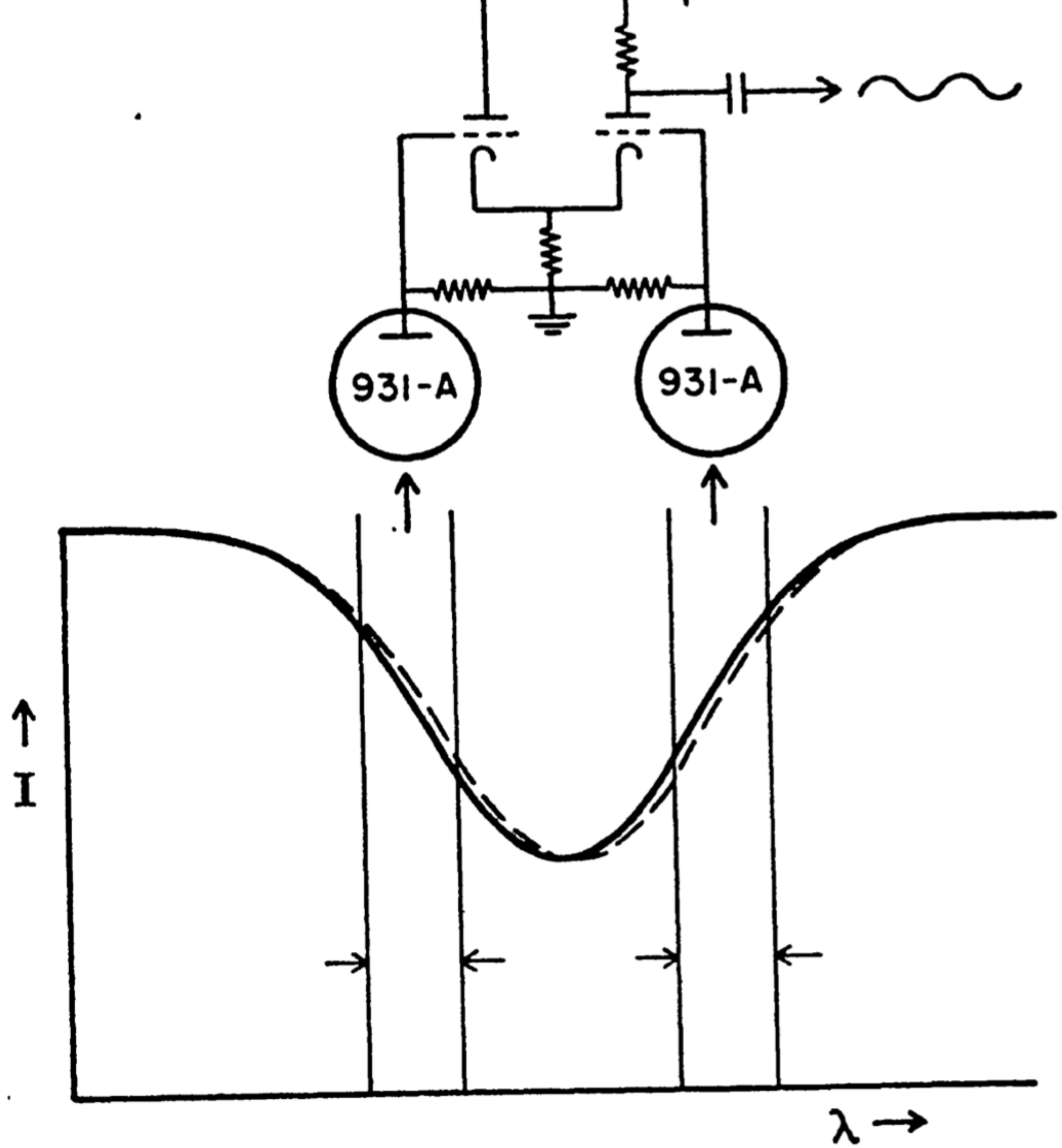
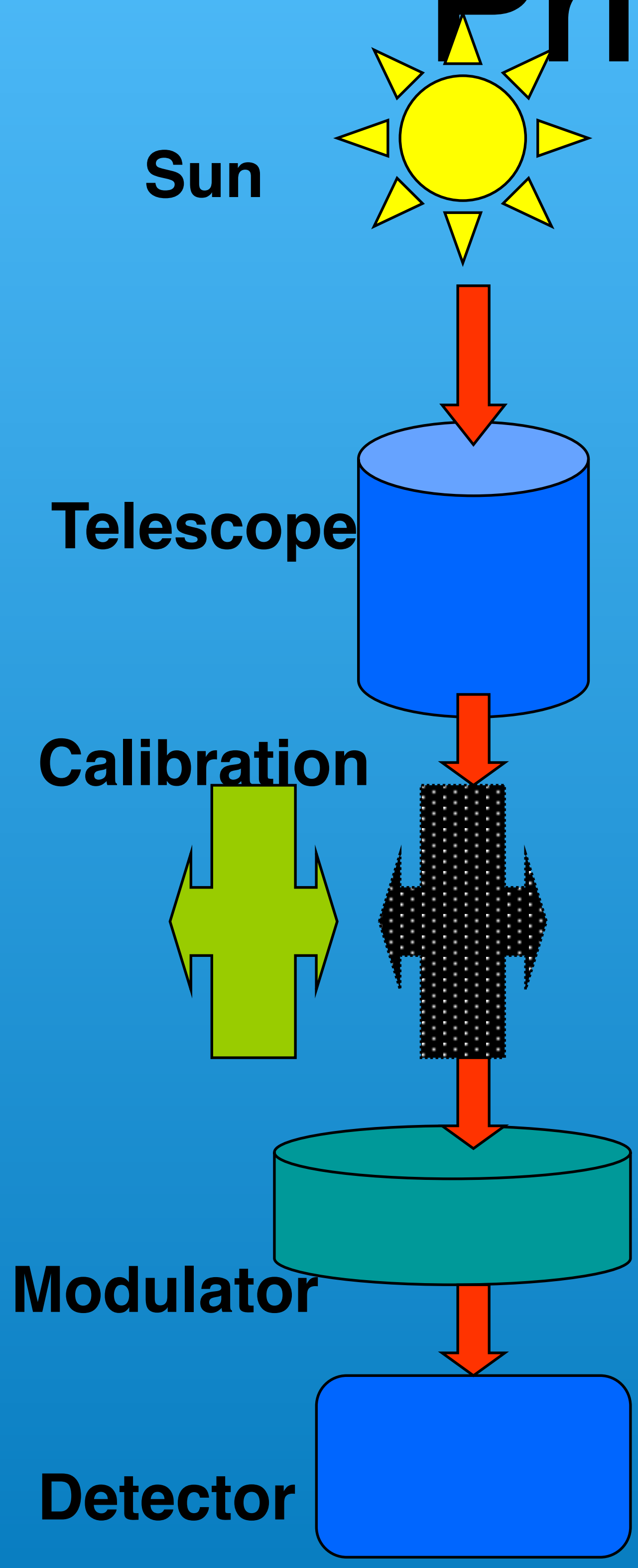


FIG. 1.—The profile of a solar line, having a half-width of about 0.1 A, is represented for the two states of circular polarization due to the Zeeman effect. The shift of the line between the two positions, in response to the alternating analyzer, occurs at a frequency of 120 cycles per second. With the line λ 5250.216 the amplitude is about 0.0008 A for a field of 10 gauss. The two slits on the wings of the line transmit light to the two photomultipliers, which are connected to a difference amplifier.

Principle of Polarimetry



- ✰Retarders and polarizers are important components
- ✰The polarization modulator varies the polarization state in a controlled and known way
- ✰Detectors can only measure intensity
- ✰ Intensity measurements are combined according to the modulation scheme, to derive the polarization of the incoming light
- ✰ Calibration is important!!
- ✰ Polarimeters differ mainly by the polarization modulation method

Optical components needed for polarimetry

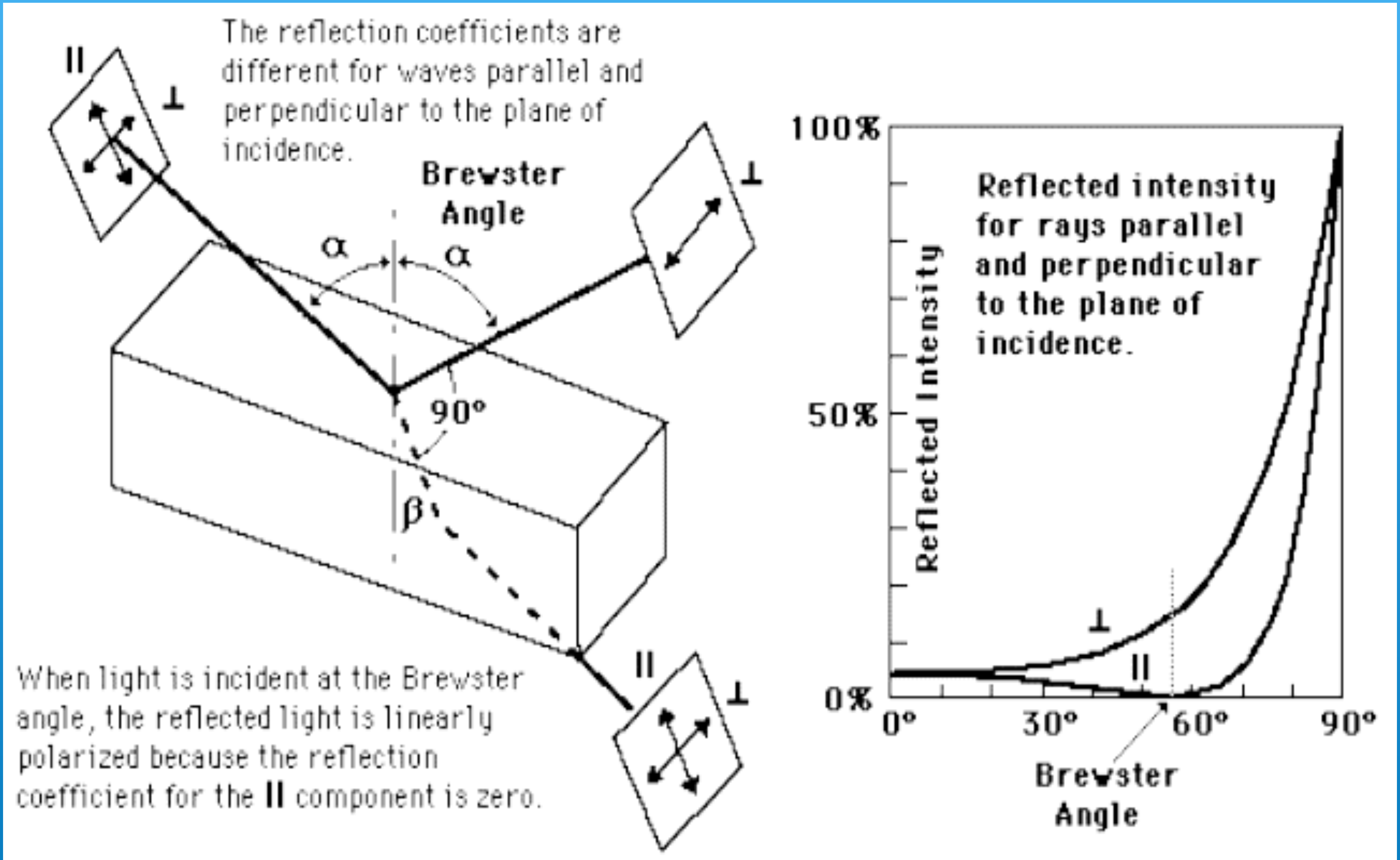
Polarizer: (Polaroid foil, birefringent materials)

Retarder: Quartz plates

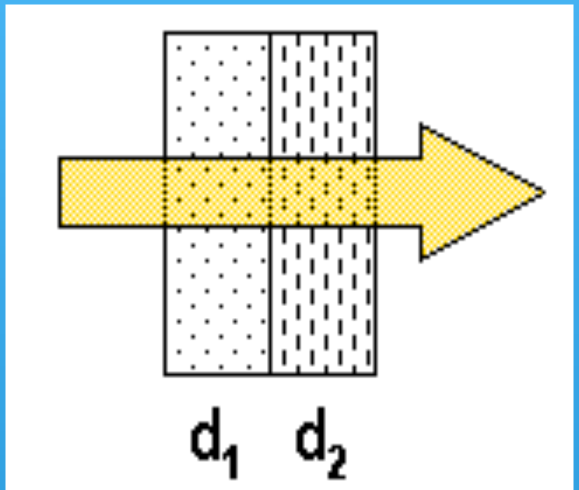
Rotator: (optically active materials, Faraday-Effekt)

Depolarizer: would be nice to have, but not existing!

Polarisation by Reflection



Quartz-Retarder



„Wave-plate“

$$\frac{\lambda}{4} = d_1(n_a - n_0) + d_2(n_0 - n_a)$$

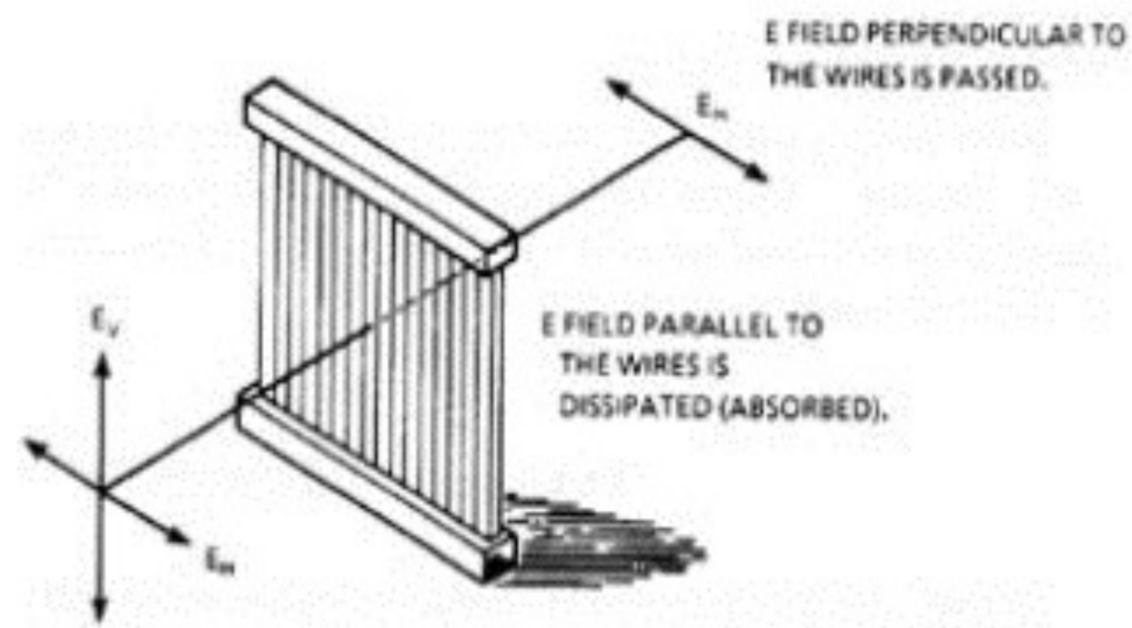
A quarter-wave retarder for 500 nm is only 15 μm thick, i..e, instable.

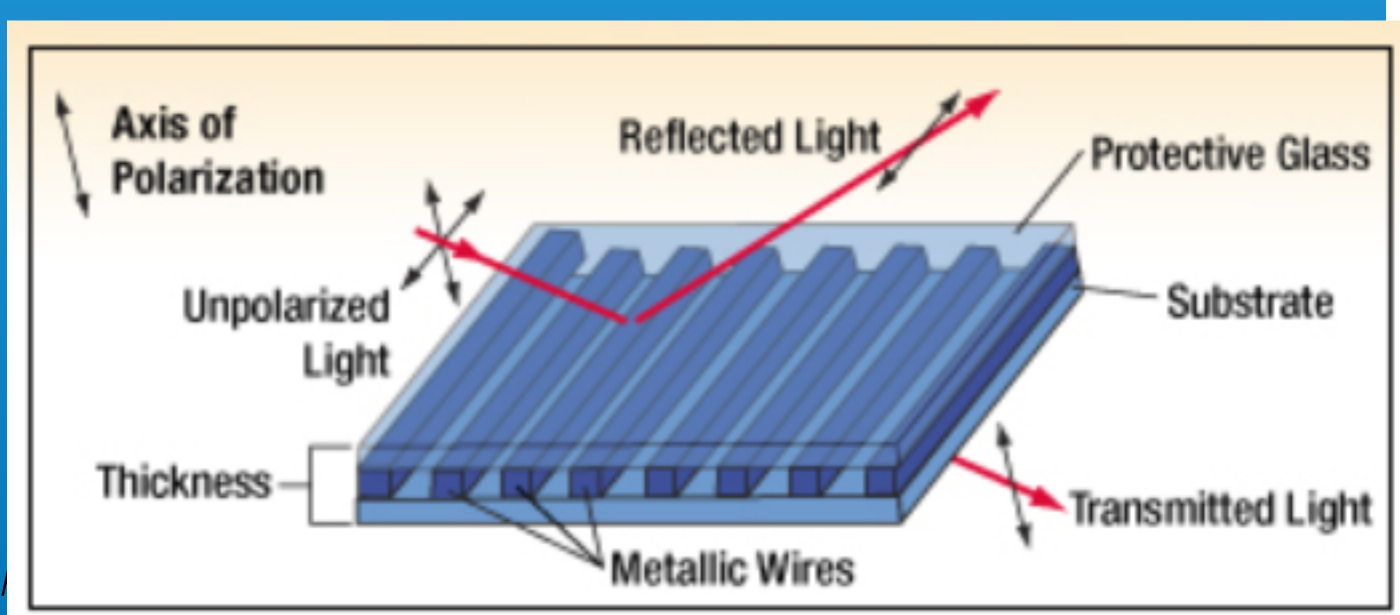
Zero order retarder are put on a glass carrier.
Composite retarder consist of 2 ca. 1mm thick plates, with a thickness difference of $\lambda/4$ (or whatever fraction of λ), in optical contact (nor air gap).

Wire-grid Polarizer

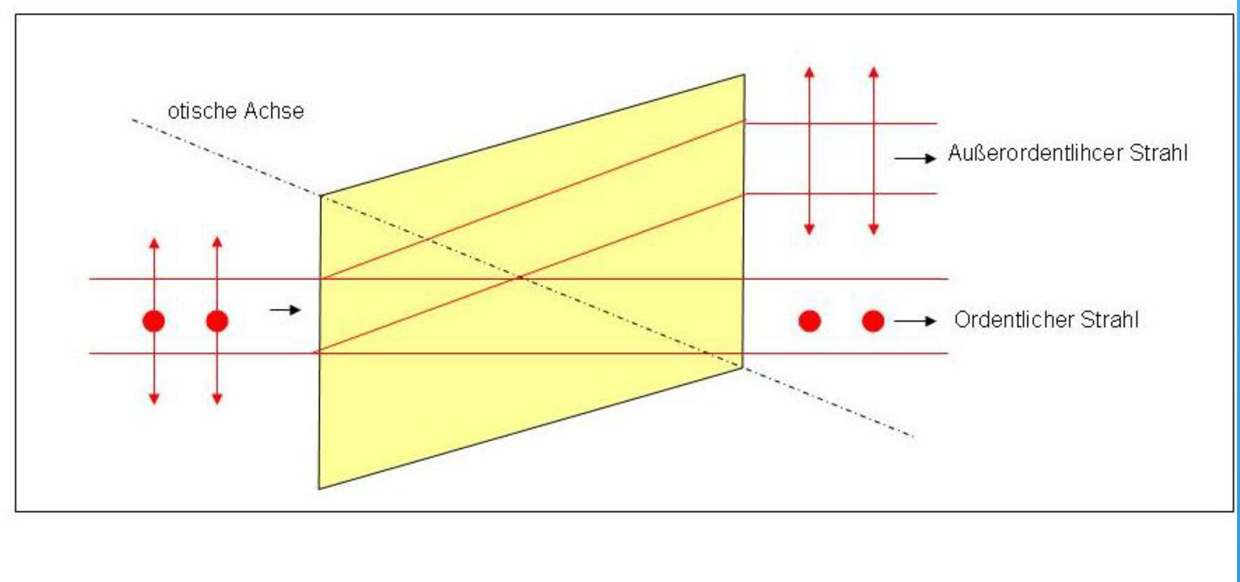
✫Grid of electrically conductive micro-wires (on a glass substrate)

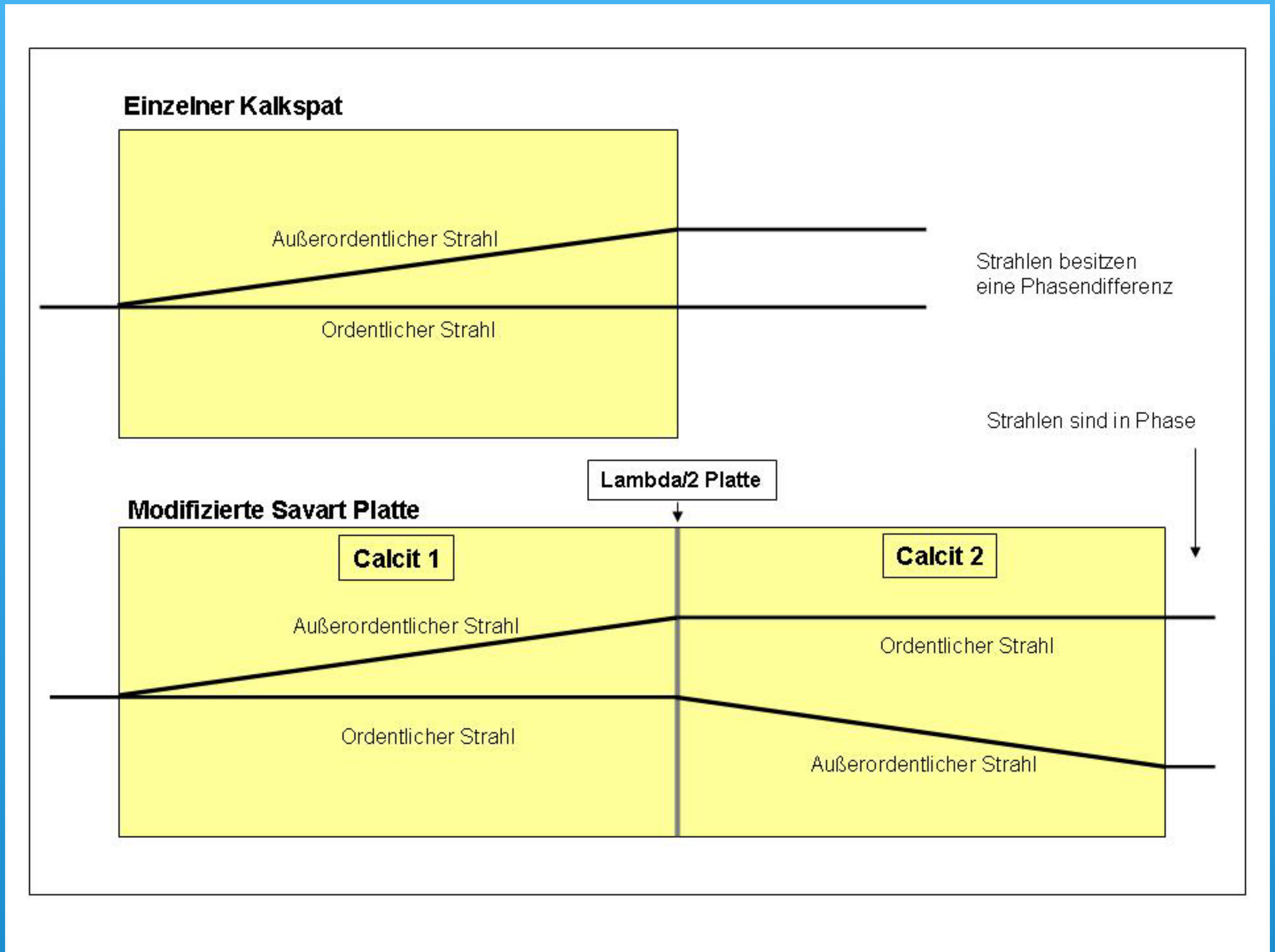
✫ Intuitively one would think that the polarization with E ll to the wires will be transmitted, but E-field parallel to wires gets weakened by induced current





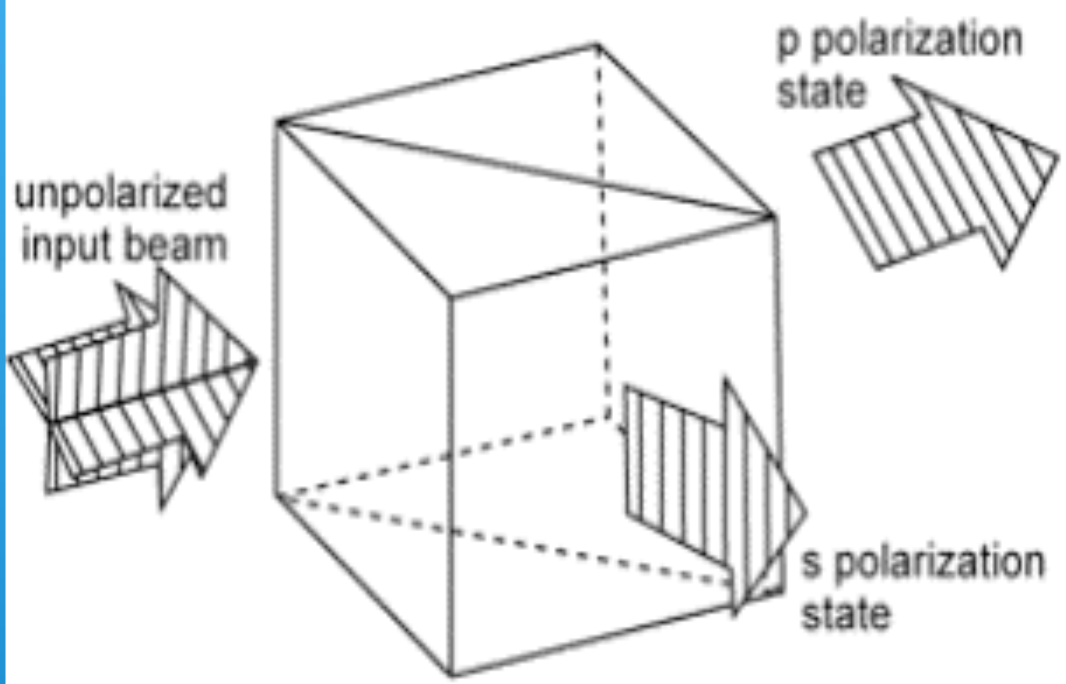
Calcite beam splitter

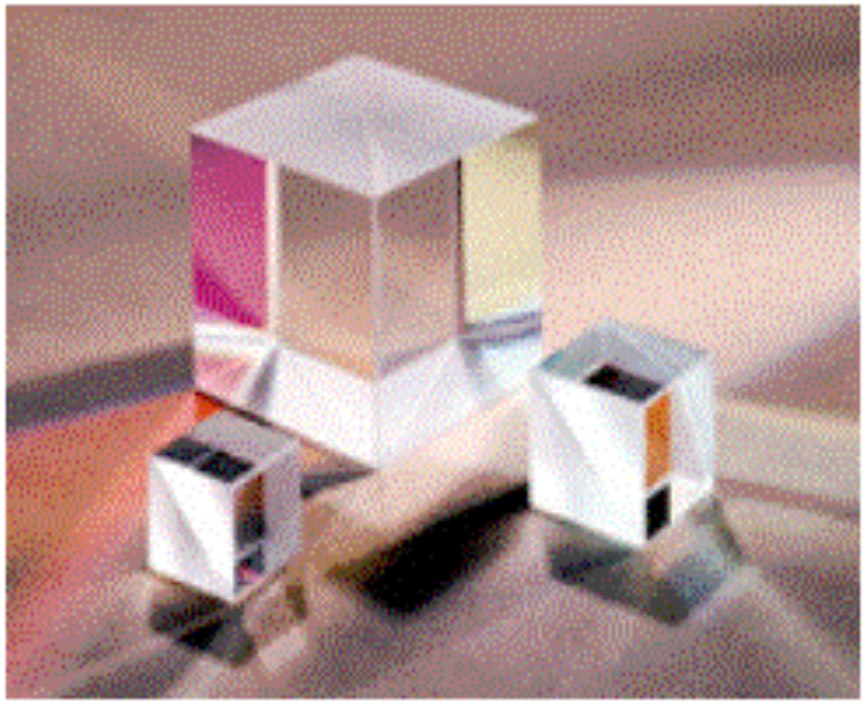




- ✰**Single piece of calcite ($CaCO_3$)**
- ✰ **Optical axis at 45 deg to the light bundle**
- ✰ **Beam separation in the visible: 0.095*L**
- ✰ **Beams are displaced parallel to each other**
- ✰ **The 2 beams are linearly polarized, perpendicular to each other**
- ✰ **The displaced Strahl has a longer optical path compared to the other one, leading to a focus difference.**
- ✰**Remedy: Use 2 calcite in series, rotated by 45 degrees. The o-beam in the first calcite becomes the eo beam in the second one, and vice versa**

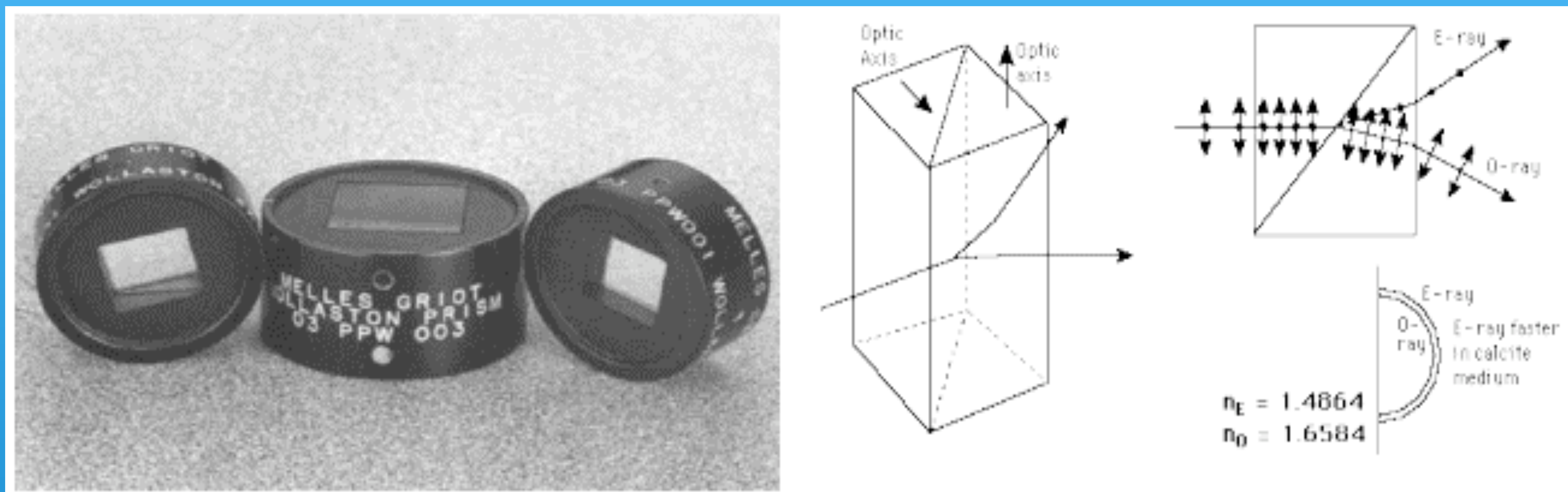
Polarizing beam splitter





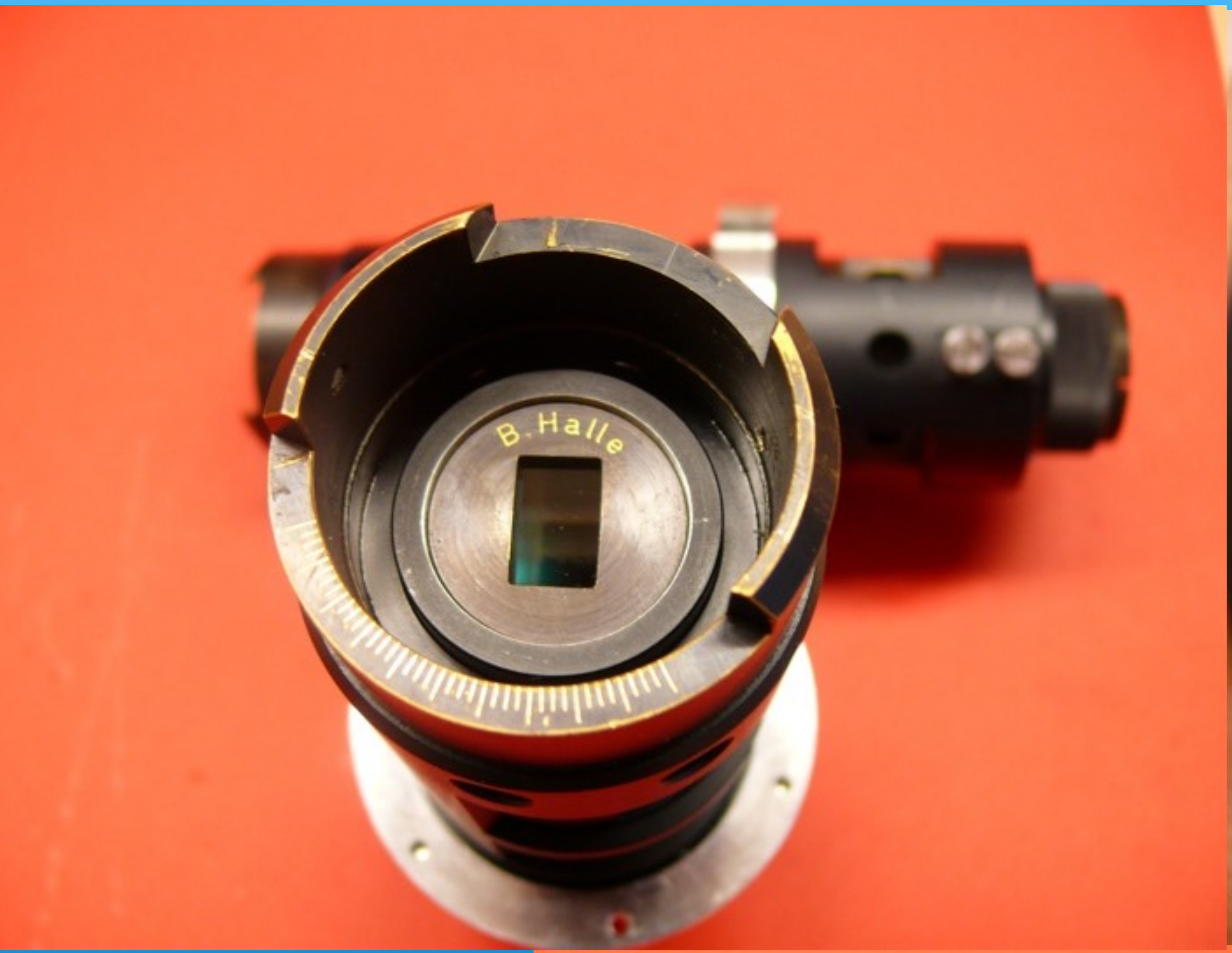
- ✰ Suitable thin films between the prisms
- ✰ One polarization orientation is totally reflected (Brewster angle)
- ✰Beam separation 90 deg

Wollaston-Beam splitter



- ✫Quartz or Calcite prisms, cemented together.
- ✫ Optical axes of the prisms orthogonal to each other
- ✫The ordinary beam in the 1st prism becomes the eo beam in the second one, and vice versa
- ✫Zhe o-beam is refracted toward the normal, the ao beam away from the normal
- ✫ Rays are deflected at an angle

Polarimeter at the Schauinsland Observatory



Stokes-Parameter

Definition of the Stokes parameter:

I: total intensity
Q: Intensity difference between vertically and horizontally polarized light
U: Intensity difference between light polarized at +45 and -45 degrees.
V: Intensity difference between right and left circular polarized light.

Examples:

S1={1,1,0,0}:	**vertically linear polarized light**
S2={1,-1,0,0}:	**horizontally polarized light**
S3={1,0,1,0}:	**Linear polarized light at 45 deg**
S4={1,0,0,1}:	**right-hand circular polarized light**
S5={1,0,0,-1}:	**left-hand circular polarized light**
S6={1,0,0,0}:	**unpolarized (natural) light.**

Stokes vector & Müller-Matrix

Stokes-Vektor:
correctly:
Stokes column matrix:

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} \langle E_x^2 \rangle + \langle E_y^2 \rangle \\ \langle E_x^2 \rangle - \langle E_y^2 \rangle \\ \langle 2E_x E_y \cos(\delta_y - \delta_x) \rangle \\ \langle 2E_x E_y \sin(\delta_y - \delta_x) \rangle \end{pmatrix} = \begin{pmatrix} \text{Intensity} \\ I(0^o) - I(90^o) \\ I(45^o) - I(-45^o) \\ I(RCP) - I(LCP) \end{pmatrix}$$

Degree of polarization

$$P = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

$$I^2 \geq Q^2 + U^2 + V^2$$

(„=„ for P=1)

Müller-Matrix:

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{pmatrix} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

Polarizer

Müller-Matrix of a polarizer at angle θ:

$$P_\theta = \frac{1}{2}\begin{pmatrix} 1 & \cos 2\theta & \sin 2\theta & 0 \\ \cos 2\theta & \cos^2 2\theta & \sin 2\theta \cos 2\theta & 0 \\ \sin 2\theta & \sin 2\theta \cos 2\theta & \sin^2 2\theta & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Polarizer at 90^0 and at 0^0:

$$P_{90} = \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_0 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Linear polarizer and Stokes-Vektor:

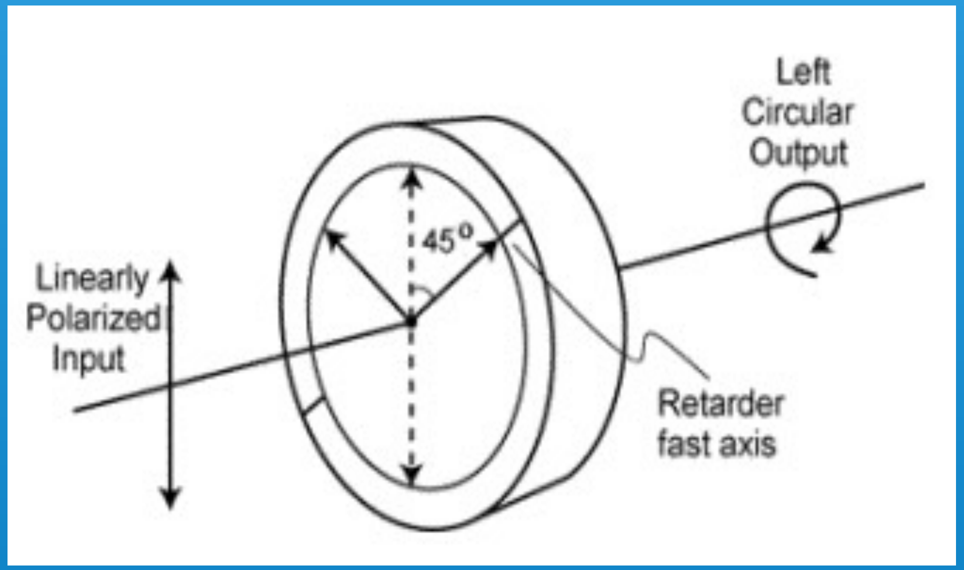
$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \frac{1}{2}\begin{pmatrix} I+Q \\ I+Q \\ 0 \\ 0 \end{pmatrix}$$

Retarder

Müller-Matrix of a retarder with retardance π/2 („λ/4-Retarder“), at angle θ :

$$P_\theta = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos^2 2\theta & \sin 2\theta \cos 2\theta & -\sin 2\theta \\ 0 & \sin 2\theta \cos 2\theta & \sin^2 2\theta & \cos 2\theta \\ 0 & \sin 2\theta & -\cos 2\theta & 0 \end{pmatrix}$$

A λ/4 retarder converts circular polarization to linear, and vice versa.



Rotator

Natural rotators: optically active substances (sugar).
Faraday-Effekt (light propagation || to a magnetic field)).

Rotation does not change the intensities

Important practical use:
Measure Mueller Matrix of rotated optical elements.

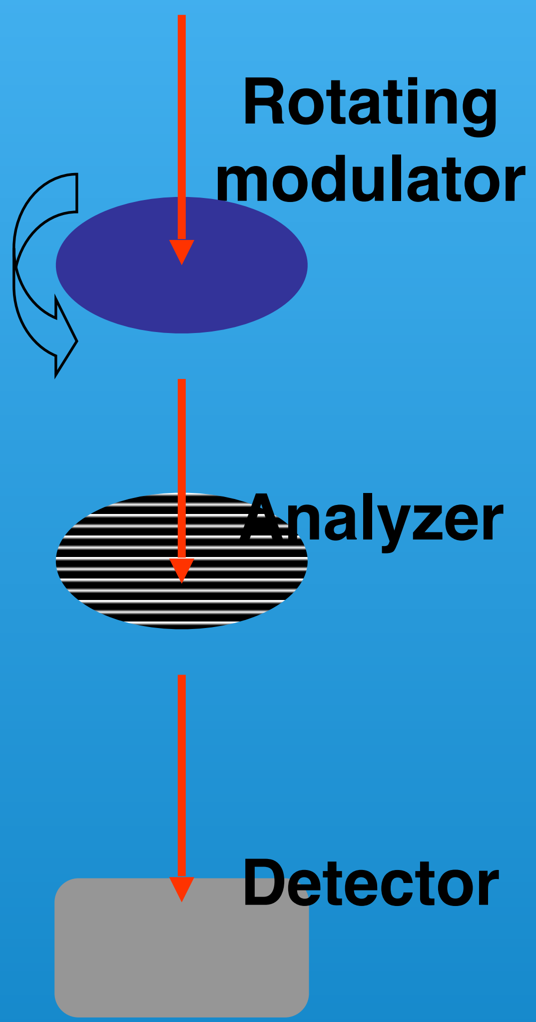
$$M_{rot}(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & \sin 2\phi & 0 \\ 0 & -\sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Examples

$$M = P_0 \cdot P_{90} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \frac{1}{2}\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Crossed polarizers: No light is transmitted!

Rotating retarder (I)



The „Advanced Stokes Polarimeter (ASP) at the Dunn Solar Telescope (Sacramento Peak) and the POLIS instrument at the VTT used a rotating modulator and a polarizing beam splitter (BS) as analyzer.
Several instruments at the DKIST will also use such a device.

Note: The VTF at the DKIST will use a polychromatic FLC + a polarizing BS

$$\begin{pmatrix} I' \\ Q' \\ U' \\ V' \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & C^2 + S^2\cos\delta & SC(1-\cos\delta) & -S\sin\delta \\ 0 & SC(1-\cos\delta) & S^2 + C^2\cos\delta & C\sin\delta \\ 0 & S\sin\delta & -C\sin\delta & \cos\delta \end{pmatrix}\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix}$$

$$S = \sin 2\varphi \quad C = \cos 2\varphi$$

Rotating retarder (II)

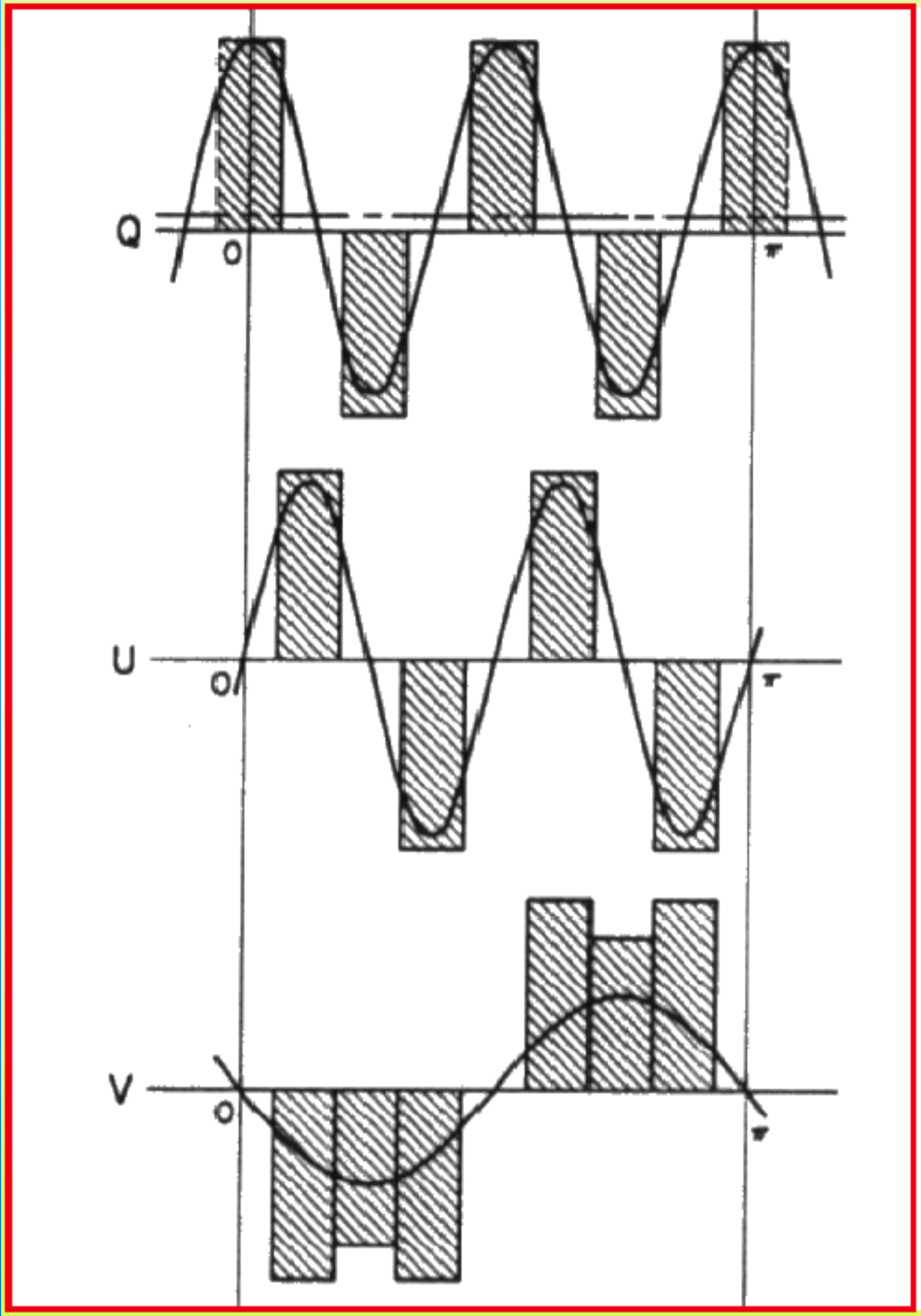
The measured intensity I' at the detector is a function of retardance δ and the angle ϕ of the rotating waveplate:

$$I'(\phi, \delta) = \frac{1}{2}(I + \frac{Q}{2}((1 + \cos\delta) + (1 - \cos\delta)\cos 4\phi) + \frac{U}{2}(1 - \cos\delta)\sin 4\phi - V \sin\delta \sin\phi)$$

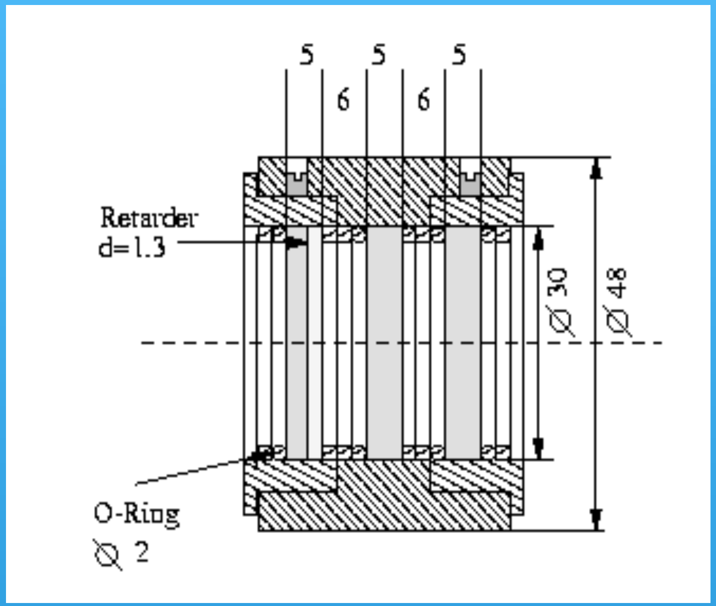
From the equation we find:

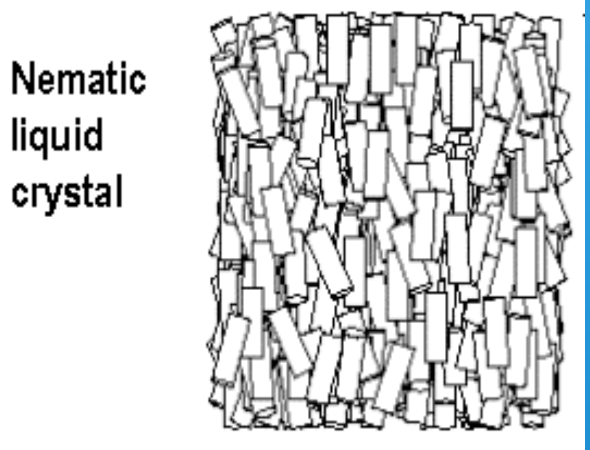
- An angle of δ=127° is needed to modulate Q, U and V with the same amplitude
- The terms that contain the rotation angle ϕ lead to a modulated signal
- The modulation frequency for U and Q is 2 times higher than for V
- There is a phase shift between Q and U is 90 degrees: measurements at 8 angles are needed
- In practice, several measurement cycles are added to increase SNR

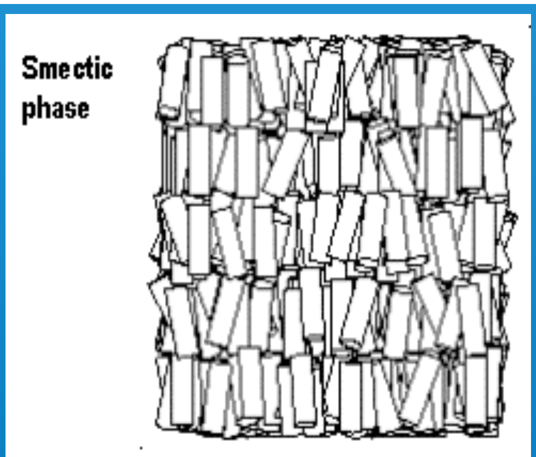
Modulation with 127^0 retarder



Variable Modulators







- ☆ Rotating waveplates (POLIS, DKIST)
- ☆ Electro-optical modulators (Kerr-cells, Pockels-cells)
- ☆ Piezo-elastic modulators (40 kHz)
- ☆ Liquid crystals (LCs):
  - ☆ Nematic LCs: molecules in random locations, but equally oriented (switch time 20 ms); example: LCDs
  - ☆ Smectic LCs: Molecules in well-defined layers, very mobile (switch time 0.15 ms); example: ferro-electric liquid crystals, FLCs

Magnetic field of a sunspot

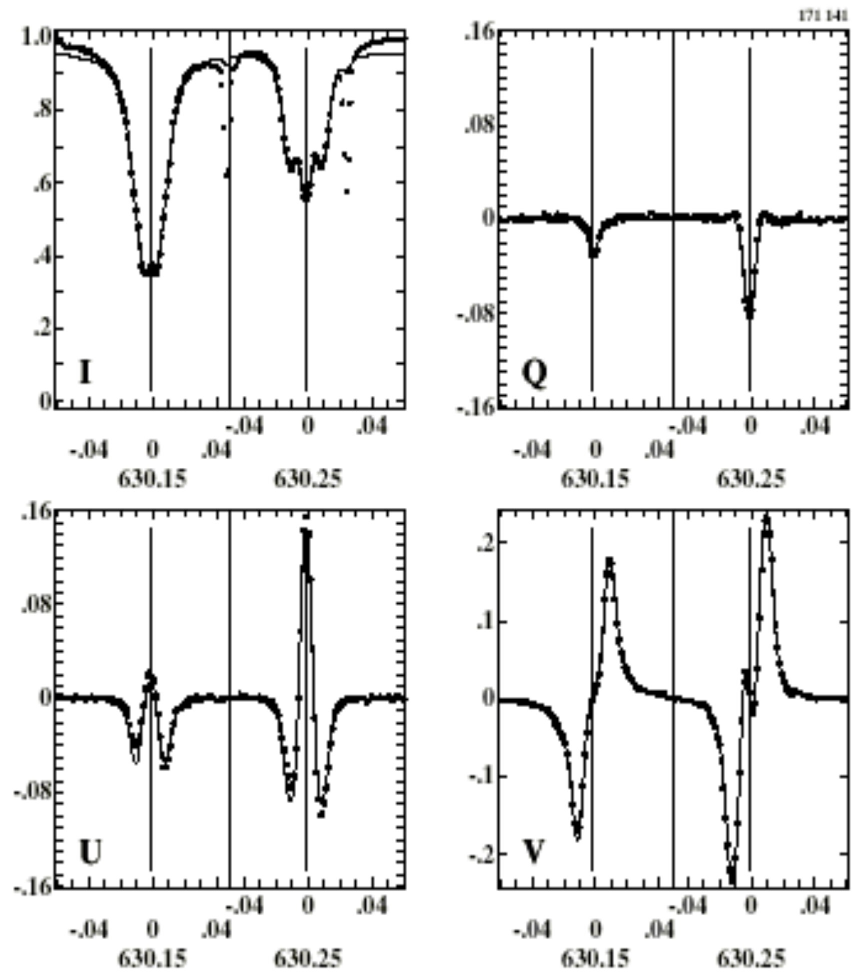
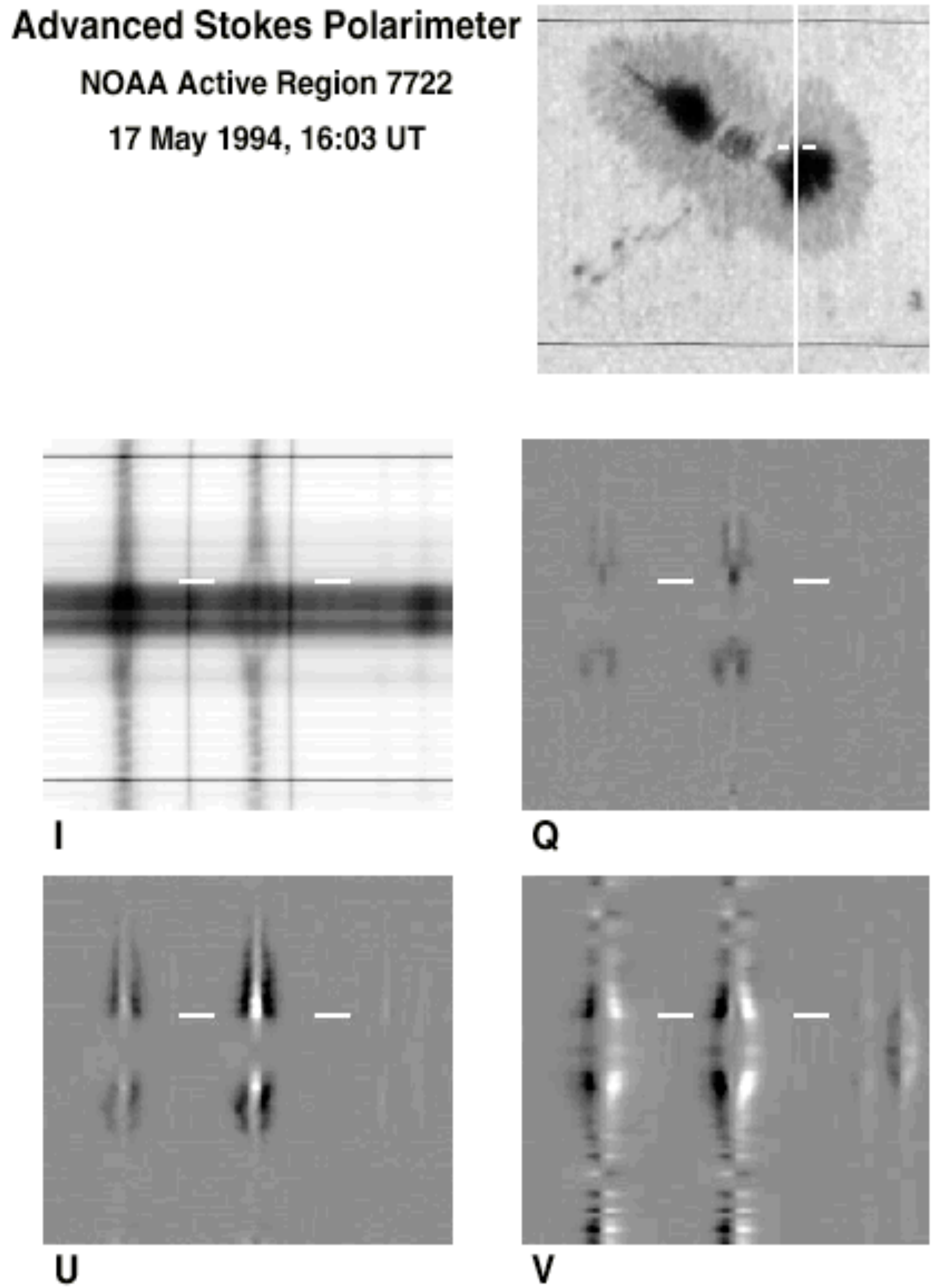
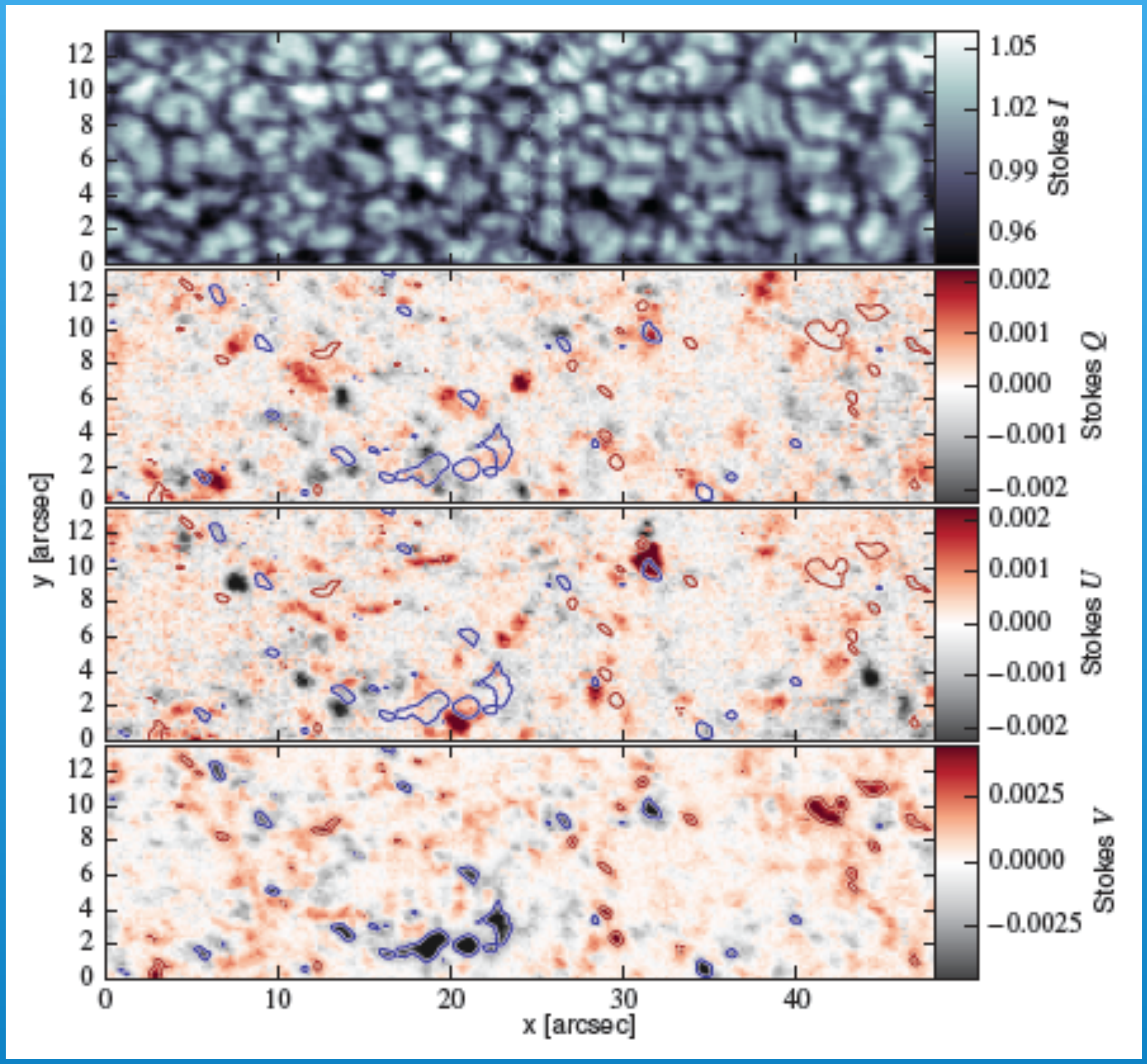


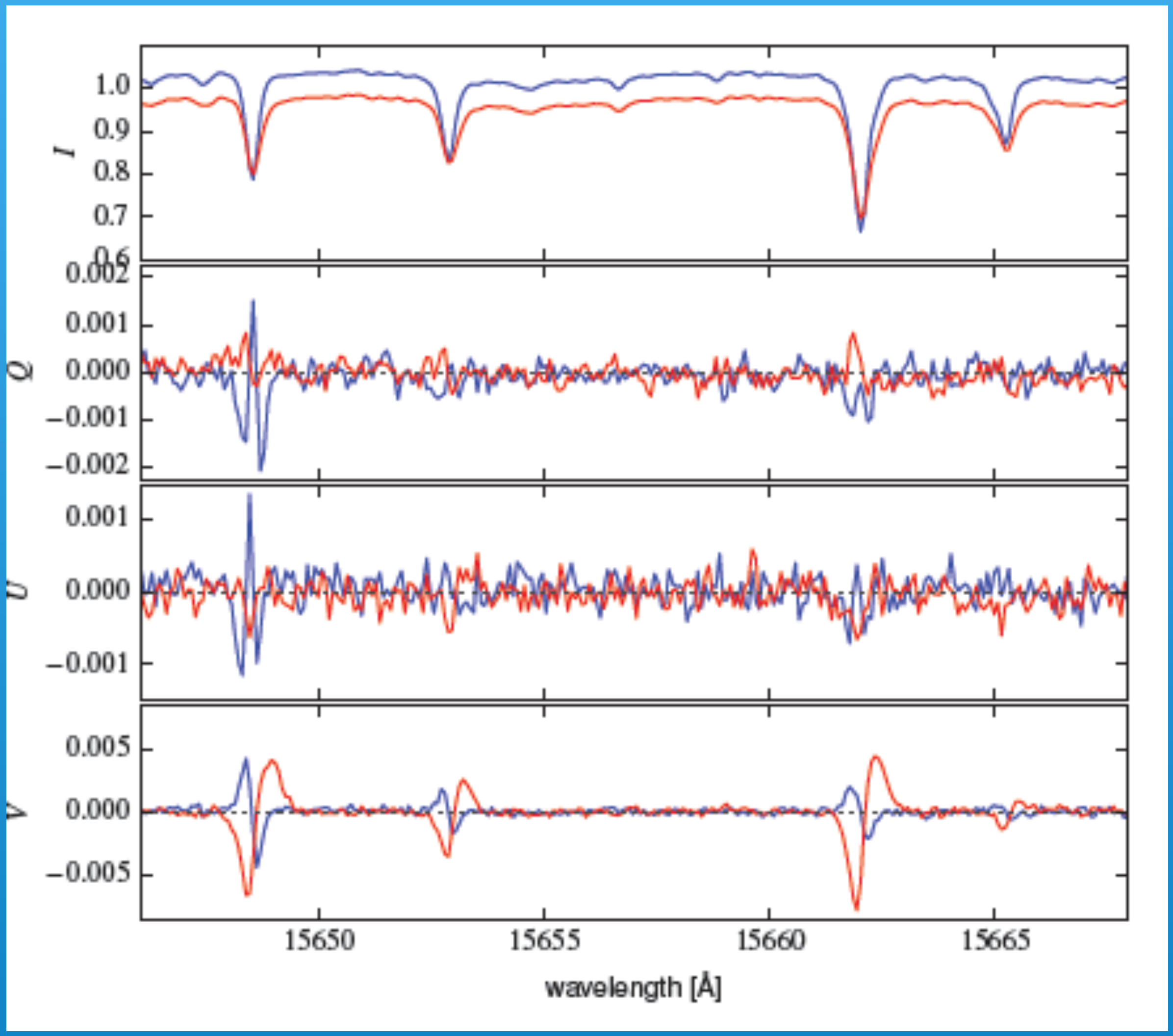
Figure 4. Polarization profiles and least-squares fits. A set of individual Stokes spectra from the location indicated in figure 1 are shown (dots), along with least-squares fits to the two solar spectrum lines (solid curves). The wavelength scale is indicated in nanometers. The extracted magnetic field parameters for this location in the sunspot are $|B| = 2206$ G, $\phi = 140^\circ$, $\gamma = 134^\circ$.



from: Lites (2001)

Quiet Sun magnetic field





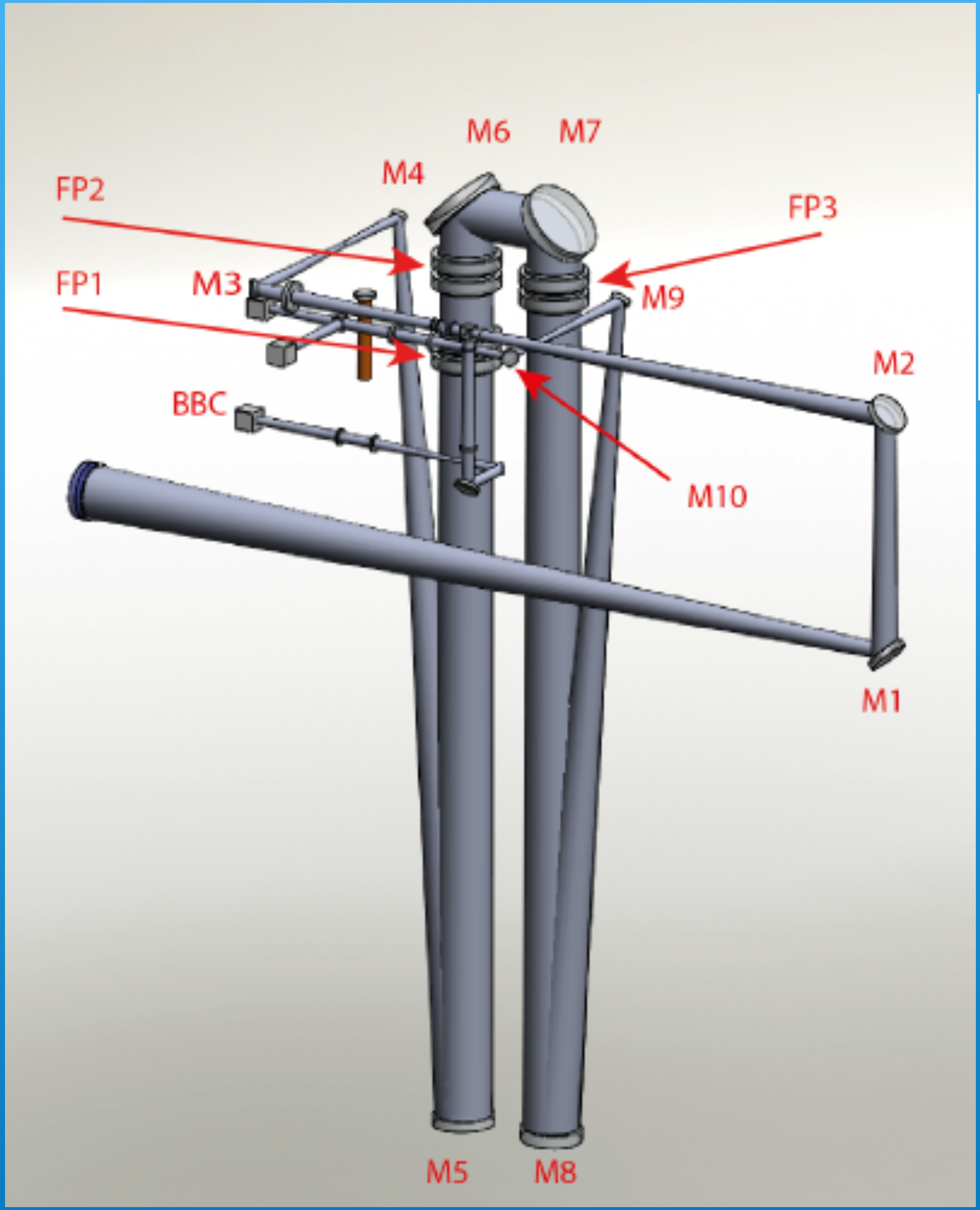
GRIS@GREGOR (Lagg et al., 2016)

The Visible Tunable Filter

- ✳ Fabry-Perot-based filter spectro-polarimeter
- ✳ Uses polychromatic FLCs as polarization modulator.
- ✳ Designed for diffraction-limited observations (according to DKIST Science Requirements)
- ✳ Spectral resolution reduced to 100,000 (6 pm at 600 nm) to increase S/N
- ✳ Polarization-free instrument design

See VTF short description

Polarimetry with the VTF



$$M_{VTF} = \begin{pmatrix} 1 & 3 \cdot 10^{-18} & 0 & 0 \\ -3 \cdot 10^{-18} & 1 & 0 & 0 \\ 0 & 0 & 1 & -1.1 \cdot 10^{-16} \\ 0 & 0 & -1.1 \cdot 10^{-16} & 1 \end{pmatrix}$$

Optical layout of the VTF with its 10 reflections. For the computations, we used bare silver coatings (n=0.135, k=3.985), other coatings will change the total transmittance of the instrument, but the elements of the normalized Mueller matrix of the VTF will not change. The resulting Mueller Matrix, computed from the reflections only, is a perfect diagonal matrix.

$$M_{VTF\_r10} = \begin{pmatrix} 1 & 0.0006 & 0.0035 & 0.0012 \\ -0.0006 & -0.9404 & -0.3185 & 0.1147 \\ -0.0035 & 0.3185 & 0.9472 & -0.0019 \\ -0.0012 & 0.1147 & 0.0188 & 0.9927 \end{pmatrix}$$

Mueller Matrix of instrument

$$M_{instr} = \begin{pmatrix} I \to I & Q \to I & U \to I & V \to I \\ I \to Q & Q \to Q & U \to Q & U \to V \\ I \to U & Q \to U & U \to U & V \to U \\ I - V & Q \to V & U \to V & V \to V \end{pmatrix}$$

Magnetic fields on the Sun are usually derived from Stokes measurements. Measurements of the Stokes vector require a polarimeter to produce known polarization states and a detector that measures the intensity of these states. Instrumental errors caused by the telescope and the instrument are usually described by Mueller matrices.

The terms I → Q,U,V, describe the polarization induced by the instrument, the terms Q ⟷ U, U ⟷ V, Q ⟷ V are called cross-talk, and Q,U,V → I is depolarization produced by the instrument

$$M_{M1M2} = \begin{pmatrix} 1 & -0.0203 & 0 & 0 \\ -0.02030 & 1 & 0 & 0 \\ 0 & 0 & 0.99 & -0.1399 \\ 0 & 0 & 0.1399 & 0.99 \end{pmatrix}$$

Mueller Matrix of DKIST M1 + M2 (off-axis telescope)

The photon dilemma of high-resolution solar physics

1. The spectral flux (photon flux per wavelength band and per m^2) from the Sun is constant.
2. The photon collection are increases with the square of the telescope aperture
3. The area of a diffraction-limited pixel (=„resolution element“) decreases with the square of the telescope aperture
4. **The number of photons per wavelength band per resolution element is independent of the telescope aperture.**
5. The characteristic time scales decrease inversely proportional to the telescope aperture -> i.e., T**he larger the telescope, the smaller the resolved spatial scale, the faster the pixel crossing time, …**
6. **Shorter integration time are needed to keep up with the faster processes on the Sun, but (4) still holds!**
7. **Example: ->>VTF**

Solar spectral flux

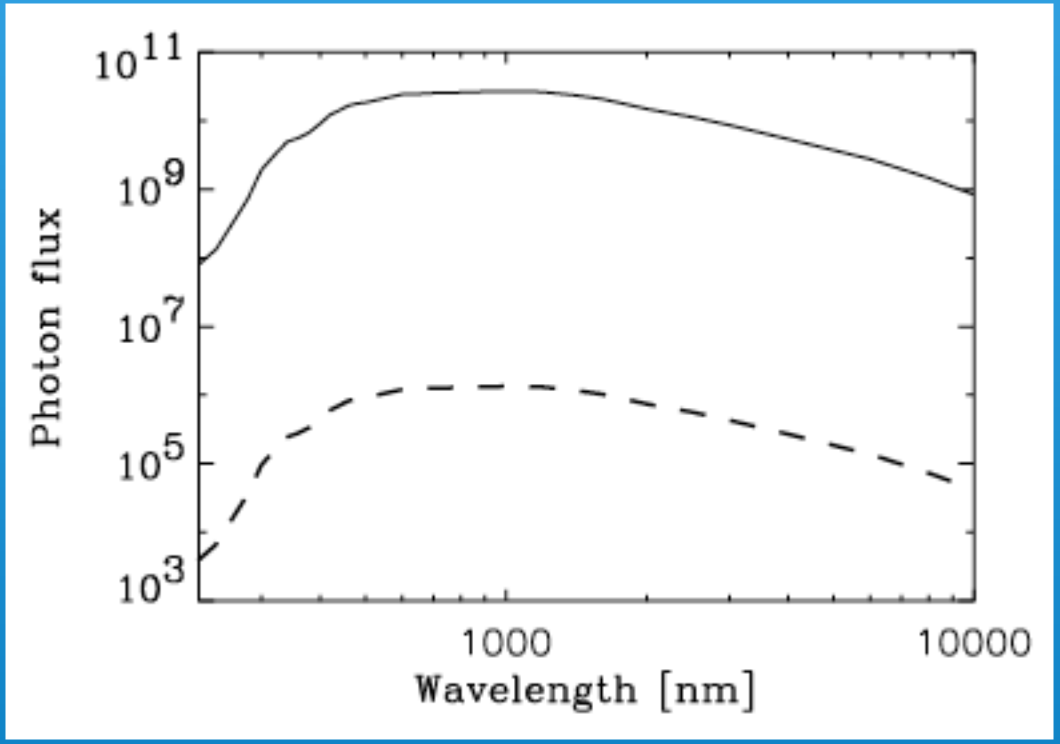
$$f_{\rm res} = \frac{\pi}{4} \frac{f_{\rm p} \lambda^2}{R_\odot^2}$$

Photon flux per resolution element of a telescope, where **f**p is the (spectral) photon flux, **λ** the wavelength and **R⊙** the angular radius of the Sun (4.65 mrad). Any increase in light-collecting area is exactly compensated by a corresponding decrease of the resolution element, therefore, the equation does not contain any instrument-dependent variable.

f_λ	1.96 J m^{-2} s^{-1}
f_p	5.4 10^{18} ph m^{-2} nm^{-1} s^{-1}
f_*	1.0 10^{8} ph m^{-2} nm^{-1} s^{-1}

The spectral flux of the Sun measured outside the Earth's atmosphere peaks in the green wavelength region. At a wavelength of 550 nm it is $f_\lambda = 1.96$ J m^{-2} s^{-1}. This corresponds to a photon flux of fp = 5.4 Å 10^{18} photons m^{-2} nm^{-1} s^{-1}.

W. Schmidt, 2001 Encyclopedia



Photons per resolution element of a telescope, f_{res}, for a wavelength interval of 1 nm and telescope transmission=1. The dashed curve shows the same function for a telescope efficiency of 0.05 and a wavelength band of 1 pm.

Photon flux comparison: TESOS & VTF

Quantity	Unit	TESOS	VTF
Wavelength	nm	617,00	617,00
Solar spectral flux at 1 AU	W m-2 Sun-1 nm-1	1,70	1,70
Distance to Sun	AU	1,00	1,00
Atmospheric transmission	(for ground-based tel)	0,90	0,90
Solar spectral flux at telescope	W m-2 Sun-1 nm-1	1,53	1,53
Photon flux at 1 AU, groundbased	**ph m-2 nm-1 sec-1**	**4,78E+18**	**4,78E+18**
Angular radius of Sun at 1 AU		960,00	960,00
Disk area	arcsec^2	2895206	2895206
Telescope aperture (0.7; 4.0)	m2	0,3848	12,566
	arcsec	0,0860	0,014
Spatial pixel size y	arcsec	0,0860	0,014
Telescope transmission		0,3127	0,390
Coudé table transmission		1,0000	0,900
Instrument transmission		0,2600	0,300
Polarizing Beam splitter		0,4900	0,490
Spectral resolution element	nm	0,0025	0,006
Photon flux at detector	ph pix-1 s-1	**9,55E+05**	**1,26E+06**
Quantum efficiency (Photonmax)		0,8000	0,560
Electrons at Detector	e s-1 pix-1	7,64E+05	7,06E+05
Integration time	s	0,025	0,025
Electrons per readout per pixel		**1,91E+04**	**1,77E+04**

Is there a remedy?

- Give up spatial resolution; use (part of) the aperture size for flux collection. -> This is considered poor use of resources, since the telescope cost increases with $r^{2.5}$.

- Give up spectral resolution to increase the spectral flux. -> Trade-off between spectral resolution and S/N.

- For filter instruments: give up spectral coverage to speed up measurements -> limits the measurement of flow speeds and complicates (or even invalidates) spectral inversion.

- For grating instruments: Give up field of view to speed up measurements (few scan steps, use of IFU) -> difficult to measure connected areas (e.g. penumbral filaments)

How to make best use of available photons

- **Maximize telescope and instrument transmission**
  - Use Ag coatings instead of Al coatings (R increases from 92 to 98 %: looks like „only 6%“, but in fact it means: „light losses reduced by a factor of 4 (from 8 to 2 %)
  - Reflectivity of 10 mirrors:
    - Ag: $T_{Ag} = 0.98^{10} = 0.82$
    - Al: $T_{Al} = 0.92^{10} = 0.43$
- Minimize „dead time“ between exposures:
  - **Take long exposures (!!! Problems with post-facto image reconstruction!!)**
  - **Use detectors with very fast readout**

Ag mirror coatings