Large-scale Dynamo vs Small-scale Dynamo

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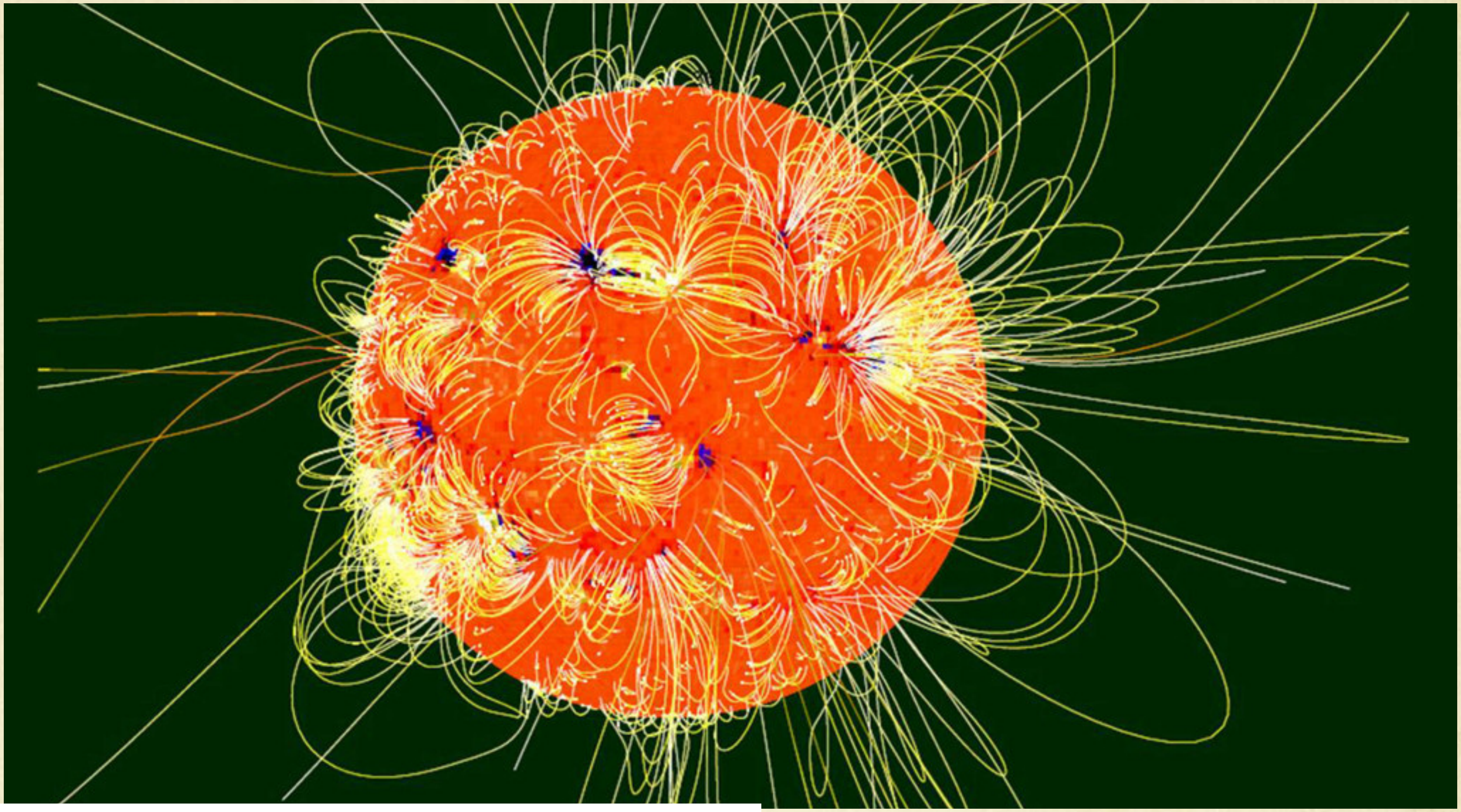
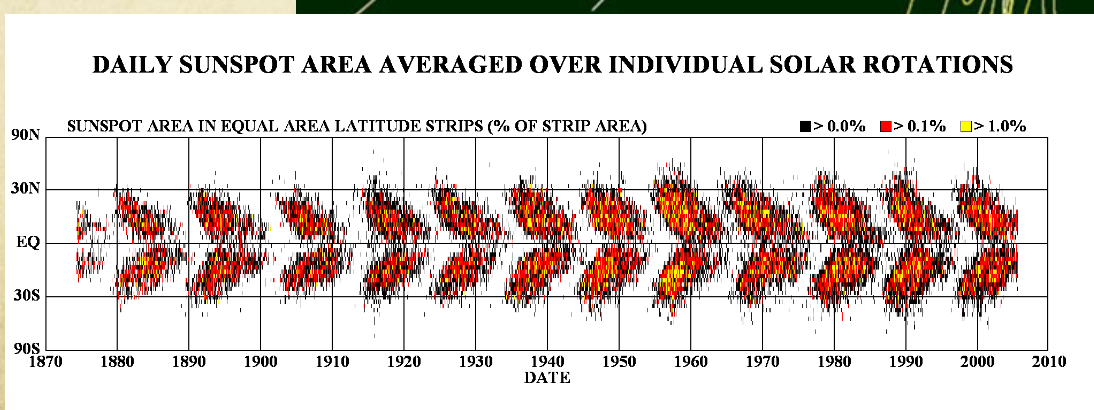
Magnetic Field of the Sun

In Space:
large-scale
structure





In Time: Coherence
(i.e. 22-years cycle)

Mean Field Dynamo Theory

Advantages:

- Filtering turns an equation with rapidly varying coefficients into ones with smoothly varying coefficients (easier to solve)
- Filtered eqs. are free of the anti-dynamo theorem

Problems:

- A given filtering may not be enough to control the fluctuations
- **Do the solution of the filtered equations coincide with the filtered solution of the full equations? ->**

-> At small Rm this is ensured as diffusion can control the growth of fields at small scales

What's going on when Rm is very high? (Rm = 10^5)

Do filtered equations mean anything at high Rm?

What is a Large-scale Dynamo? Rm=10^5

$$\frac{\partial \bar{B}}{\partial t} = \nabla \times (\bar{u} \times \bar{B}) - \frac{1}{Rm} \nabla^2 \bar{B}$$

$$\bar{u} = V_0 \cos\left(\frac{2\pi}{L_y} y\right) \hat{e}_x + helical\ flow$$

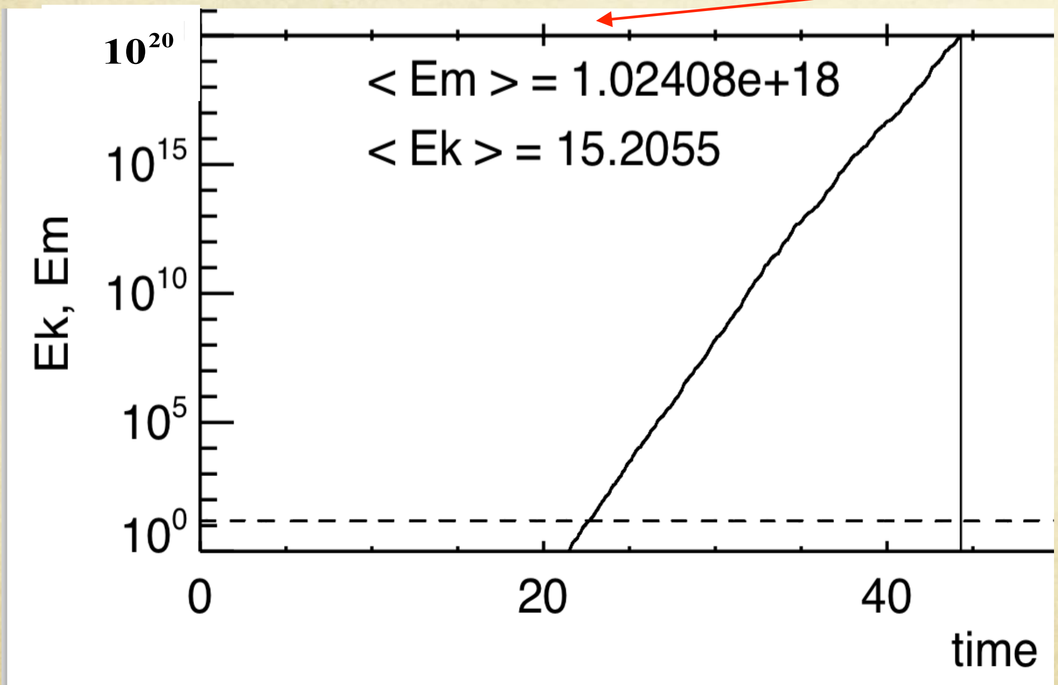
$$\boldsymbol{B} = \boldsymbol{b}(x, y, t)\ e^{ik_z z}$$

shear amplitude

Case when the shear = **5.2**

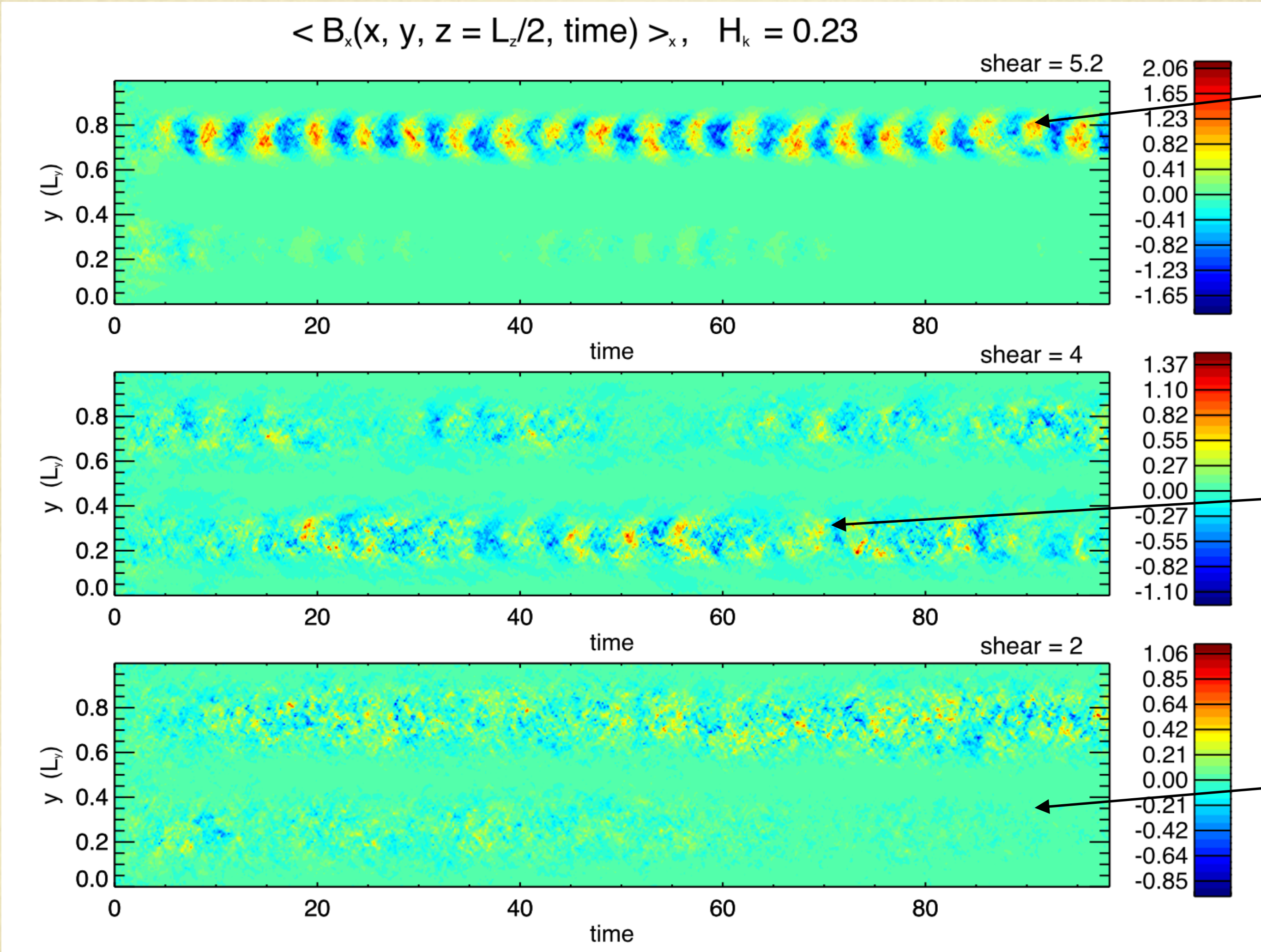
Growth rate σ, eddy-turn over time τ :

$\sigma = \mathbf{1.0}$ ^ $\tau = \mathbf{0.2}$



All components (at small and large scale) grow at the same rate: **This rate is determined by the small-scales that have been removed from the filtered equations**

Wave component when kx=0



Wave for kx =0; what about the other wave number kx≠ 0 ?

Wave ?

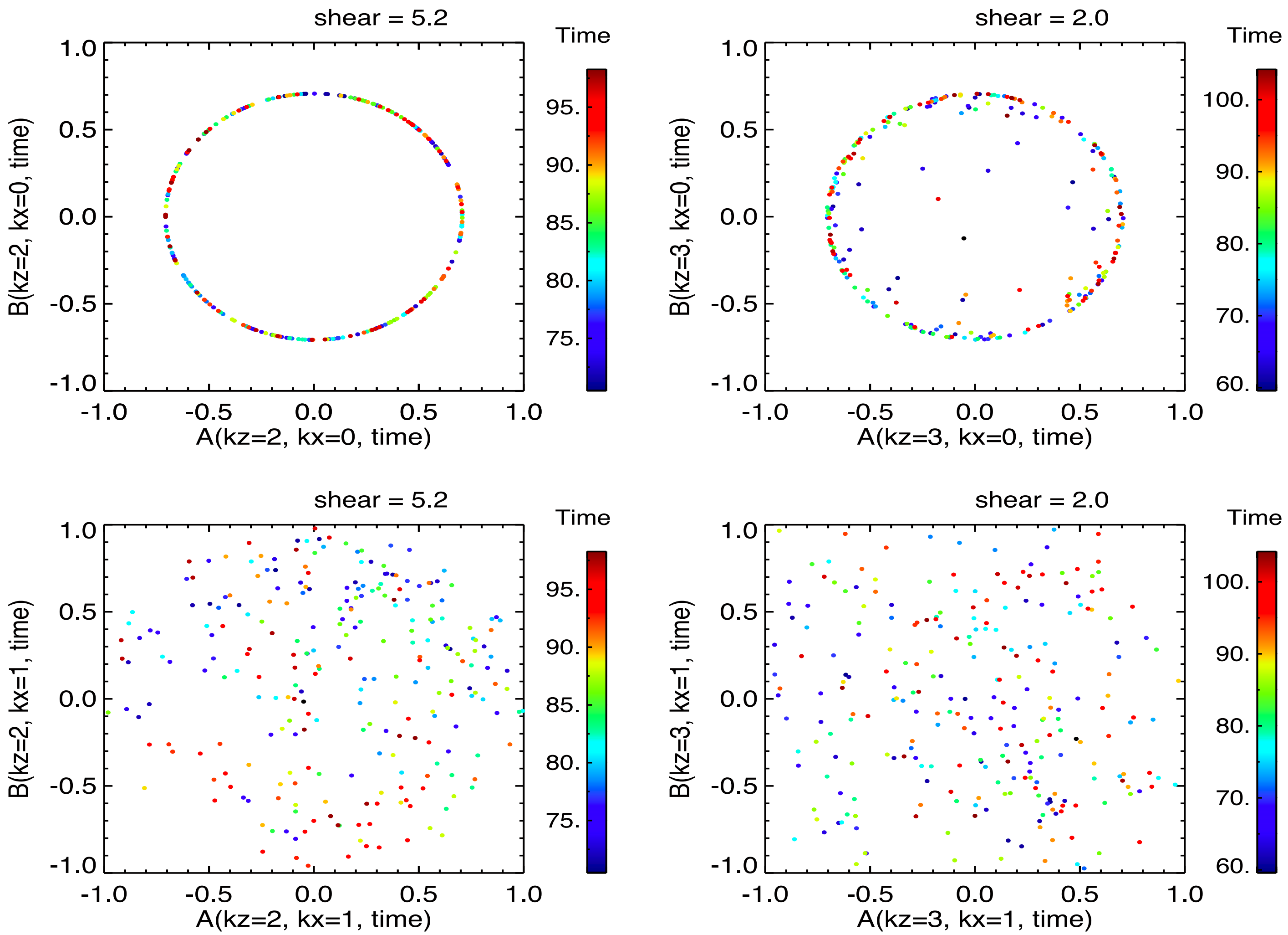
I do not distinguish any wave, but may be…

G. Nigro, P. Pongkitiwaichakul, F. Cattaneo, S.M. Tobias, **464,** L119–L123 (2017) **MNRAS**
P. Pongkitiwaichakul , **G. Nigro** , F. Cattaneo , and S. M. Tobias, **825,** 23 (2016) **ApJ**

Phase Diagrams

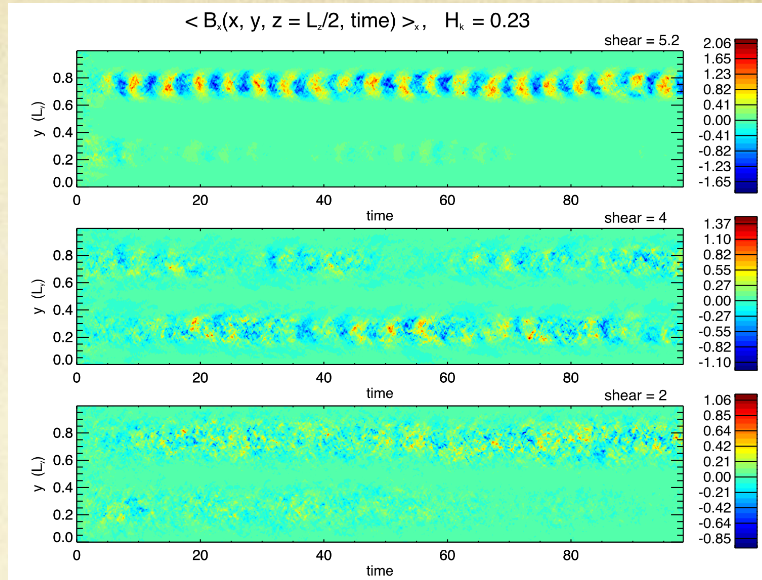
A possible wave pattern

$$scale\ \ 1/kx \ \ => \ \ FT_x\left[B_x\ (x,y,z,t)\right] = \left[A_{kx}(y,\ t)\ \sin(k_z z) + B_{kx}(y,\ t)\ \cos(k_z z)\right]\ e^{\sigma t}$$

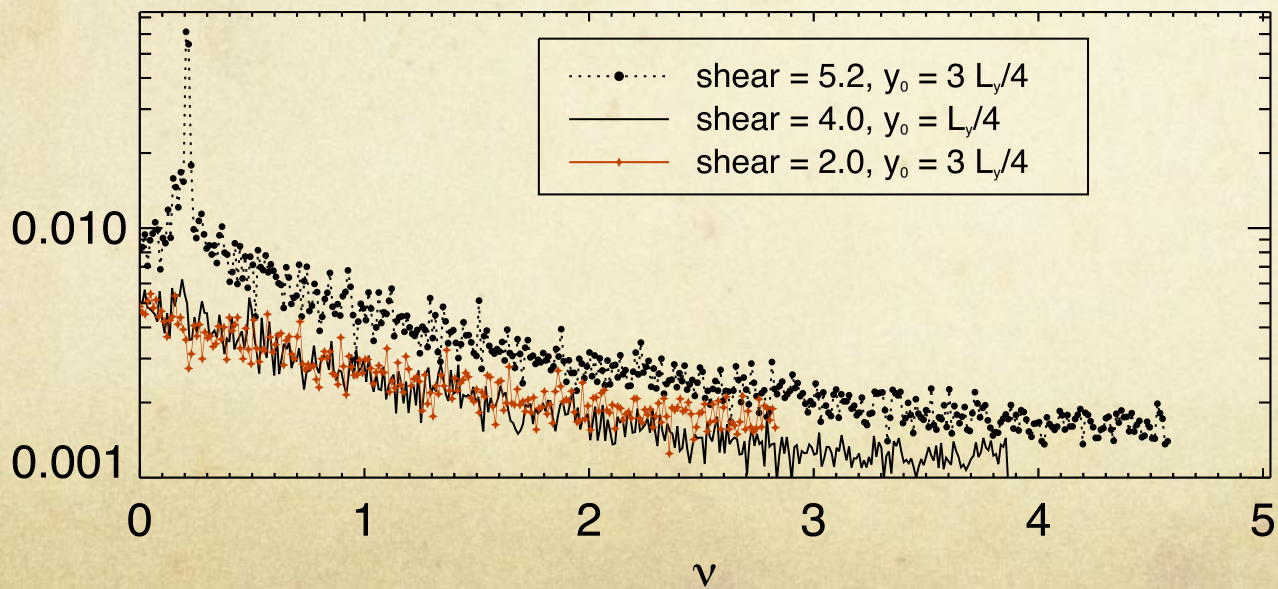


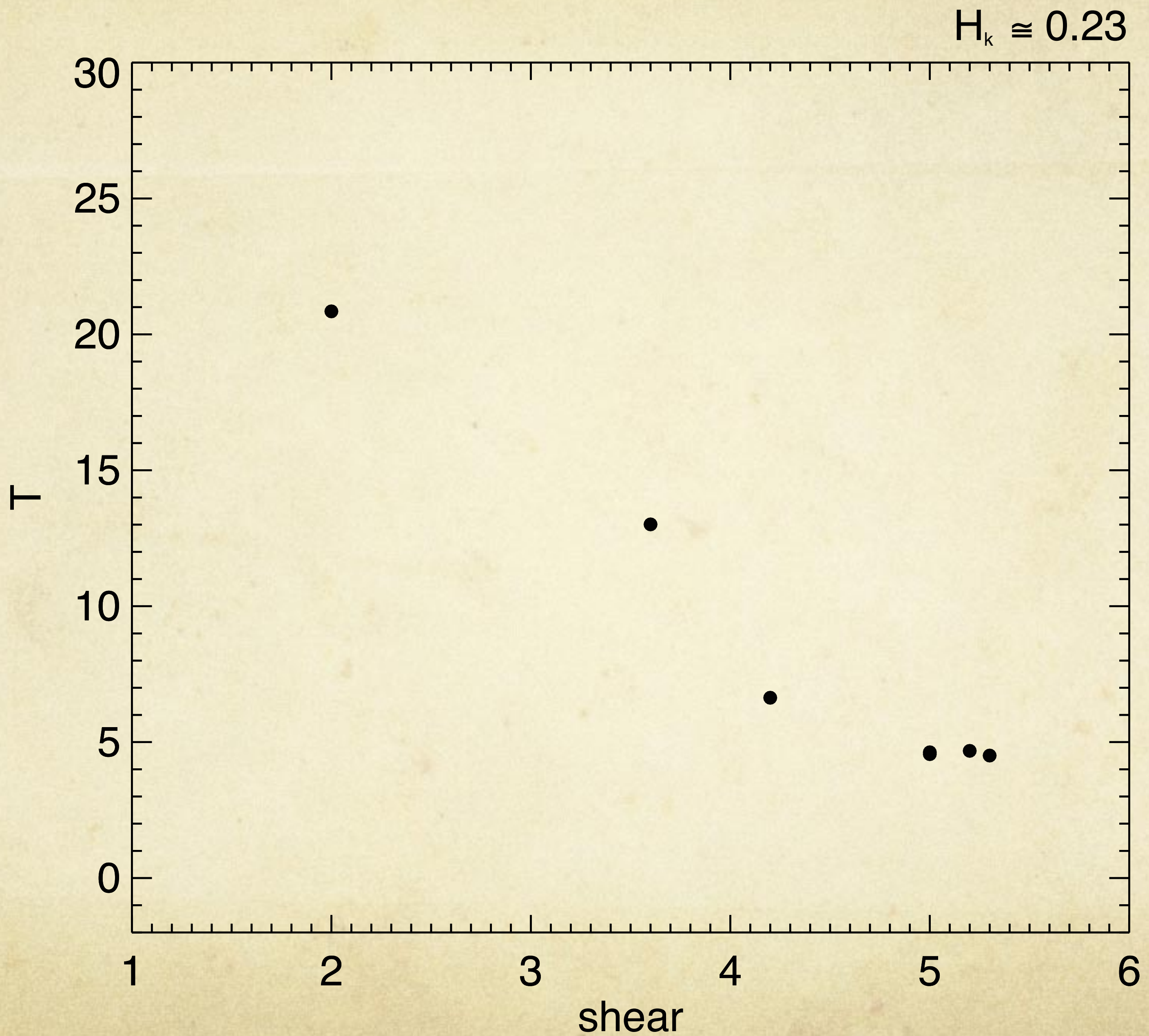
Shear = 5.2 => P = 1/ ν_{max} = 1/0.21 ~ 5
Shear = 2.0 => P = 1/ ν_{max} = 1/0.48 ~ 20

The dependence of these frequencies *does* yield periods that are compatible with those that emerge from a theory based on the solution of the filtered equations (mean-field electrodynamics), and **is not determined by the small scale dynamo**



$< |fft(B_x(x, \; y = y_0, \; z = L_z/2, \; t))|^2 >_x$





$H_k \cong 0.23$
30
25
20
15
10
5
0
T
1
2
3
4
5
6
shear

Definition of large-scale dynamo effect

- The period of the wave component is comparable with those predicted by MFE
- The wave component **does not have a separate growth rate** from the rest of the magnetic structure.
- Both small-scale and large-scale dynamo have the same source: small-scale turbulence
- The wave component can only be unambiguously identified from the rest of the structures by its **phase coherence** during the time: all the other parts of the solution are incoherent in time

It could be better to consider a ***definition of large-scale dynamo action that considers the time–scale of evolution of the pattern, rather then one that relies on spatial scales alone***

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Thank you for your time!